Answers of Practice Paper 6

Section I

Answer 1.

(a) Let the number be subtracted be x

New numbers = 23 – x; 30 – x ; 57 – x ; 78 – x

Acc. to given condition, they are in proportion;

$\frac{23 - x}{30 - x}$ = $\frac{57 - x}{78 - x}$

(23 – x) (78 – x) = (57 – x) (30 – x)

1794 – 23x – 78x + x^2 = 1710 – 57x – 30x + x^2

1794 – 101x = 1710 – 87x

1794 – 1710 = – 87x + 101x

84 = 14

x = $\frac{84}{14}$

x = $\underline{\underline{6}}$

(b) TPT: $2\tan^2\theta + \cot^2\theta = \text{cosec}^2\theta \times \sec^2\theta$

LHS: $2\tan^2\theta + \cot^2\theta$

$2 + \frac{sin^2\theta}{cos^2\theta} + \frac{cos^2\theta}{sin^2\theta}$

$\frac{2si\ ^2\theta cos^2\theta + sin^4\theta + cos^4\theta}{sin^2\theta co\ ^2\theta}$ = $\frac{(sin^2\theta + cos^2\theta)^2}{sin^2\theta\ cos^2\theta}$;

Since, $sin^2\theta + cos^2\theta$ = 1

$\frac{1^2}{sin^2\theta\ cos^2\theta}$

$\frac{1}{sin^2\theta\ cos^2\theta}$

$\text{cosec}^2\theta \times \sec^2\theta$

L.H.S = R.H.S

(c) Length of piece = x

Cost of each piece = $\frac{75}{x}$

New length = x + 2

Cost of each piece = $\frac{75}{x+2}$

According to the equation:-

$\frac{75}{x} - \frac{75}{x-2}$ = 10

$\frac{75x + 150 - 75x}{x^2 + 2x}$	=	10
150	=	$10x^2 + 20x$
15	=	$x^2 + 2x$
$x^2 + 2x - 15$	=	0
$x^2 + 5x - 3x - 15$	=	0
$x(x + 5) - 3(x + 5)$	=	0
$(x - 3)(x + 5)$	=	0
$x + 5 = 0$	or	$x - 3 = 0$
$x = -5$	or	$x = 3$

Ignoring the –ve value:-

x = 3m

Answer 2.

(a)

$$2(x - 1)(x - 5) = 5$$
$$2[x^2 - 5x - x + 5] = 5$$
$$2x^2 - 12x + 10 = 5$$
$$2x^2 - 12x + 5 = 0$$

a = 2

b = – 12

c = 5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(5)}}{2(2)}$$
$$x = \frac{12 \pm \sqrt{144-40}}{4}$$
$$x = \frac{12 \pm \sqrt{104}}{4}$$
$$x = \frac{12 \pm \sqrt{26 \times 4}}{4}$$
$$x = \frac{12 \pm 2\sqrt{26}}{4}$$
$$x = \frac{6 \pm \sqrt{26}}{2};$$
$$x = \frac{6 + \sqrt{26}}{2}$$
$$x = \frac{6 + 5.099}{2}$$
$$x = \frac{110.99}{2}$$
$$x = 5.5495$$

x = 5.55 ; or

$$x = \frac{6 - \sqrt{26}}{4};$$
$$x = \frac{6 - 5.099}{2}$$

x = $\frac{0.901}{2}$

x = 0.4504

x = {5.55; 0.45}

(b) Dimensions of a model of multi-storey building

= 1.2 m × 75 cm × 2 m

= 1.2 m × 0.75 m × 2 m

Scale factor (k) = $\frac{1}{30}$

i. model length = k × actual length

1.2 = $\frac{1}{30}$ × actual length

Actual length = 36 m

ii. model breadth = k × actual length

0.75 = $\frac{1}{30}$ × actual length

Actual breadth = 22.5 m

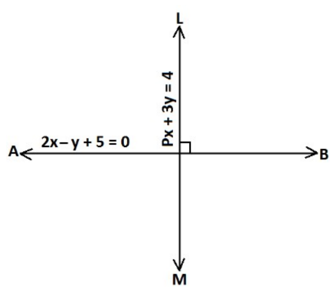
iii. model height = k × actual length

2 = $\frac{1}{30}$ × actual length

∴ Actual height = 60 m

∴ Actual dimensions = 36 m × 22.5 m × 60 m

(c)



For line AB

2x – y + 5 = 0

y = mx + c

– y = – 2x – 5

y = $\left(\frac{-2}{-1}x\right) + \left(\frac{-5}{-1}\right)$

y = 2x + 5

m = 2

For line LM

px + 3y = 4

y = mx + c

3y = – px + 4

y = $\left(\frac{-p}{3}x\right) + \left(\frac{4}{3}\right)$

m' = $\frac{-p}{3}$

m' of LM = – $\left(\frac{1}{m\ of\ AB}\right)$, perpendicular

$\frac{-p}{3} \quad = -\left(\frac{1}{2}\right)$

$\frac{-p}{3} \quad = \frac{-1}{2}$

$-2p \quad = -3$

$P \quad = \frac{-3}{-2} \quad = \frac{3}{2}$

Answer 3.

(a) $-2\frac{2}{3} \leq x + \frac{1}{3}$; $x + \frac{1}{3} < 3\frac{1}{3}$

$-\frac{8}{3} \leq x + \frac{1}{3}$; $x < \frac{10}{3} - \frac{1}{3}$

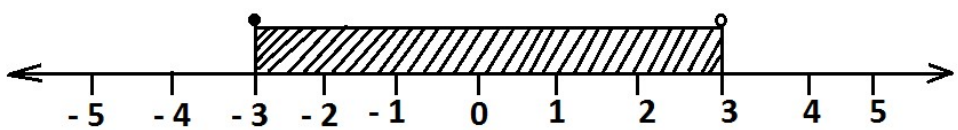
$-\frac{8}{3} - \frac{1}{3} \leq x$; $x < \frac{9}{3}$

$-\frac{9}{3} \leq x$; $x < 3$

$-3 \leq x$; $x < 3$

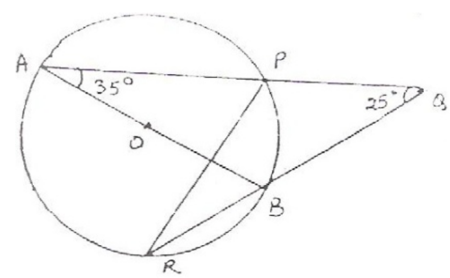
$\therefore -3 \leq x < 3$

S.S. = $\{x: -3 \leq x < 3 ; x \in R\}$



(b) Given:

i. AB is the diameter of the given circle.
ii. APQ and RBQ are two straight lines through P and B.



To Find:

i. ABQ
ii. APR
iii. AMR

Statement	Reason
(1) In ΔAQB; ∟BAP + ∟PQB + ∟ABQ = 180^0 ; ∟ABQ = 180 – (35 + 25) = 180 – 60 = 120^0	Angle sum property
(2) ∟ABR = 180 – 120 = 60^0	Linear pair
[3] ∟APR = ∟ABR = 60^0 ∟ABR = 60^0	Angle in the same segment are equal
[4] In ΔAMP ; ∟AMR = ∟MAP + ∟MPA = 35 + 60 = 95^0	Exterior angle theorem.

(c)

Number (x)	Frequency (f)	Fx	Cf
5	1	5	1
10	2	20	3
15	5	75	8
20	6	120	14
25	3	75	17
30	2	60	19
35	1	35	20
Total	20	390	

i. Mean (X) = $\frac{\sum fx}{\sum f}$ = $\frac{390}{20}$ = 19.5

ii. Median = Value of $\left(\frac{n}{2}\right)^{th}$ term

= Value of $\left(\frac{20}{2}\right)^{th}$ term ;

= Value of 10^{th} term

= 20 (highest frequency)

iii. Mode = 20 (Highest frequency)

Answer 4.

(a)

INVESTMENT 1	INVESTMENT 2
FV = Rs 100	FV = Rs 25
n = $\frac{8000}{80}$ = 100 shares	n = ?
r = 7%	r = 18%
MV(CP) = Rs 80	MV = Rs 41
MV(SP) = Rs 80	I = Rs 8700
Sale P = -	D = -
I = Rs 8000	
D = ?	

Investment 1 :-

D = $\frac{FV \times n \times r}{100}$

D = $\frac{100 \times 100 \times 7}{100}$

D = Rs. 700

Sale proceeds = 100×80 = Rs 8000

I_2 = SP + D = 8000 + 700 = Rs. 8700

n_2 = $\frac{I}{MV}$ = $\frac{8700}{41}$ = 212 share

D_2 = $\frac{FV \times n \times r}{100}$ = $\frac{25 \times 212 \times 18}{100}$ = Rs. 954

R = $\frac{(954-700)\times 100}{8000}$ = $\frac{25400}{8000}$ = $\underline{\underline{3.175\%}}$

(i) gain = 954 – 700 = Rs. 254

(ii) Annual income = Rs. 954

(iii) R = 3.175%

(b)

Given:

i. A circle with center O.

ii. PQ = QR = RS

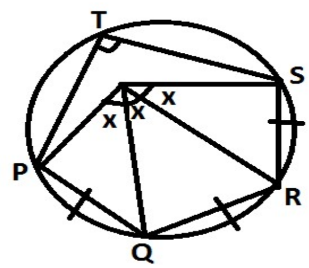
iii. ∟PTS = 75^0.

<u>To Find:</u>

i. ∟POS

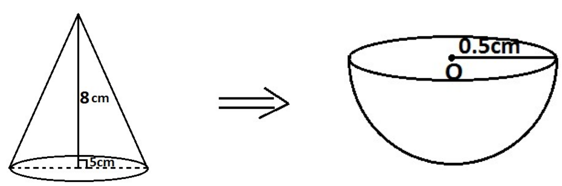
ii. ∟QOR

iii. ∟PQR



Statement	Reason
(1) ∟POS = 2∟PTS = 2 × 75 = 150^0	Angle at centre is twice that of circumference
(2) ∟POQ = ∟QOR = ∟ROS = $\frac{150}{3}$ = 50^0	Equal chord subtends equal angle at the centre.
(3) PQR = $\frac{360-2x}{2}$ = $\frac{360-2\times 50}{2}$ = $\frac{360-100}{2}$ = 130^0	Reflex angle property

(c)



Volume of cone = n × volume of sphere

n = $\frac{\text{volume of cone}}{\text{volume of sphere}}$

= $\left(\frac{1}{3}\pi r^2 h\right) \div \left(\frac{4}{3}\pi r^3\right)$

= $\frac{1}{3}\times\pi\times r^2\times h\times\frac{3}{4}\times\frac{1}{\pi}\times\frac{1}{r^3}$

= $r^2\times h\times\frac{1}{r^2}$

= $\frac{5\times5\times8\times10\times10\times10}{5\times5\times5\times4}$

= $\frac{16\times10\times10}{4}$

= $\frac{16\times100}{4}$

= $\frac{1600}{4}$

= <u>400 sphere are formed</u>

Section II

Answer 5.

(a) x = Rs.400

n = 36 months

r = 11%

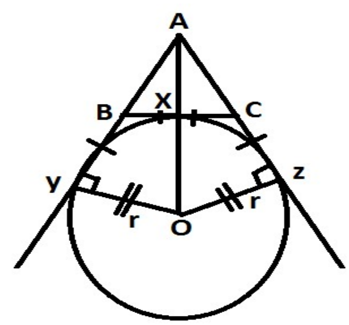
MV = $nx + \left(\frac{nxr\,(n+1)}{2400}\right)$

= $(36 \times 400) + \left(\frac{400\ \times 36 \times 37 \times 11}{2400}\right)$

= 14400 + 2442

= Rs. 16842

(b)



Given:

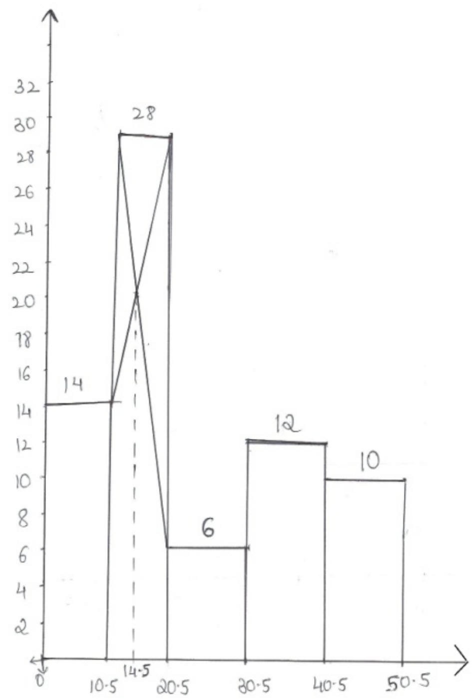
i. A circle touches the side BC of a triangle ABC at X

ii. AB and AC produced at Y and Z respectively.

To Prove That: AY is half the perimeter of triangle ABC.

Statement	Reason
(1) AY = AZ (2) BX = BY (3) CX = CZ	Tangents from a point outside the circle are equal
(4) OXB = OXC = 90^0	Tangent is perpendicular to centre of O
(5) In ΔAXB and ΔAXC; AX = AX , ∟B = ∟C ∟AXB = ∟AXC ΔAXB ≅ ΔAXC	Common side Parts of equal tangents From statement 4 above
(6) BX = CX	CPCT
(7)AY = AB + BY	
(8) AY = AB + BX	From statement 2
(9) AZ = AC + CZ = AC + CX	From statement 3
(10) AB + BC + CA = Perimeter AB + BX + XC + CA = P AY + AZ = P 2AY = P AY = $\frac{P}{2}$	From statement 1 Hence proved.

(c) Graph:



Answer 6.

(a) A = $\begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}$

B = $\begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix}$

A + B = $\begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix}$ = $\begin{bmatrix} -3+1 & 2+a \\ 2+b & -4 \end{bmatrix}$ = $\begin{bmatrix} -2 & 2+a \\ 2+b & -4 \end{bmatrix}$

A – B = $\begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix}$ = $\begin{bmatrix} -3-1 & 2-a \\ 2-b & -4 \end{bmatrix}$ = $\begin{bmatrix} -4 & 2-a \\ 2-b & -4 \end{bmatrix}$

(A + B) (A – B) = $\begin{bmatrix} -2 & 2+a \\ 2+b & -4 \end{bmatrix} \begin{bmatrix} -4 & 2-a \\ 2-b & -4 \end{bmatrix}$;

= $\begin{bmatrix} 8+(2+a)(2-b) & -4+2a-8-4a \\ -8-4b-8+4b & (2+b)(2-a)+16 \end{bmatrix}$;

A² = $\begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 2 & -4 \end{bmatrix}$ = $\begin{bmatrix} 9+4 & -6-8 \\ -6-8 & 4+16 \end{bmatrix}$ = $\begin{bmatrix} 13 & -14 \\ -14 & 20 \end{bmatrix}$;

B² = $\begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix} \begin{bmatrix} 1 & a \\ b & 0 \end{bmatrix}$; = $\begin{bmatrix} 1+ab & a+0 \\ b+0 & ab+0 \end{bmatrix}$ = $\begin{bmatrix} 1+ab & a \\ b & ab \end{bmatrix}$;

A² – B² = $\begin{bmatrix} 13-1-ab & -14-a \\ -14-b & 20-ab \end{bmatrix}$;

= $\begin{bmatrix} 8+(2+a)(2-b) & -12-2a \\ -16 & (2+b)(2-a)16 \end{bmatrix}$ = $\begin{bmatrix} 12-ab & -14-a \\ -14-b & 20-ab \end{bmatrix}$

By equality of matrices;

$-12 - 2a = -14 - a$

$a = 2$

$-16 - 14 - b$

$-2 = -b$

$b = 2$

(b) n × V(lead shots) = ¼ V(water over flows) ;

$n \times \frac{4}{3}\pi r^3 = \left(\frac{1}{3}\pi r^2\right) \times ¼$

$n \times 4 \times (0.5)^3 = ¼ \times \frac{1}{3} \times \pi \times (5)^2 \times 8$

$n \times 4 \times (0.5)^3 = \frac{5^2 \times 8^2}{4}$

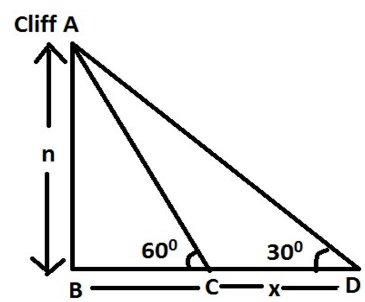
$n = \frac{5^2 \times 2}{4 \times (0.5)^3}$

$n = 100$

(c) Time = 6 mins = 60 × 6 = 360 secs ;

d = x m ;

$s = \frac{d}{t} = \frac{x}{360}$ m/s ;



In ΔABC;

$\tan 60^0 = \frac{AB}{CB}$;

$\sqrt{3} = \frac{h}{a}$;

$h = a\sqrt{3}$ --------- (i)

In ΔABD;

$\tan 30^0 = \frac{AB}{BD}$;

$\frac{1}{\sqrt{3}} = \frac{h}{x + a}$;

$x + a = h\sqrt{3}$; substituting from equation (i) ;

$x + a = (a\sqrt{3})\sqrt{3}$

$x + a = 3a$

$x = 2a$

Total distance = 2a + a = 3a ;

BD = 3a meter ;

BC = a m ;

CD = n = 2a ;

If 2a ; 6min ; a ; ? ;

$\frac{a \times 6}{2a} = 3$ min; or

$s = \frac{2a}{360}$;

d = a ;

t = $\frac{d}{s}$

= $\frac{a}{\frac{2a}{360}}$;

$\frac{360a}{2a}$ = 180 sec = $\frac{180}{60}$ = 3 min

Answer 7.

(a) Let the number to be added = a

$\therefore$ 2x + 7 = 0 (factor)

2x = − 7

x = $\frac{-7}{2}$

f(x) = 0

f $\left(\frac{-7}{2}\right)$ = 0

f(x) = $2x^3 + 5x^2 - 11x - 10 + a$

f $\left(\frac{-7}{2}\right)$ = $2\left(\frac{-7}{2}\right)^3 + 5\left(\frac{-7}{2}\right)^2 - 11\left(\frac{-7}{2}\right)^3 - 10a + a$

0 = $2 \times \frac{-34}{8} + 5 \times \frac{49}{4} + \frac{77}{2} - 10 + a$

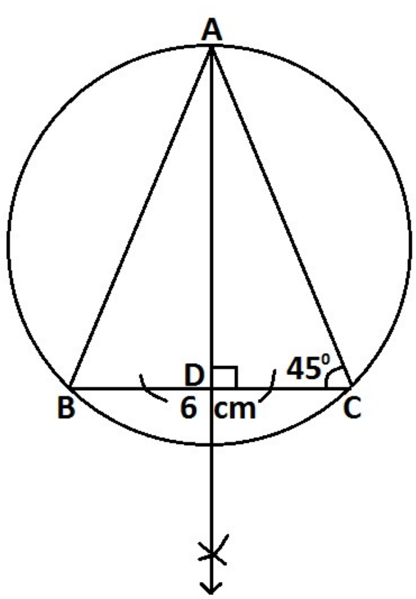
0 = $\frac{-343 + 245 + 154 - 40}{4} + a$

0 = $\frac{16}{4} + a$

0 = 4 + a

a = − 4

(b) Construction:-



Radius = ______

(c) Given:

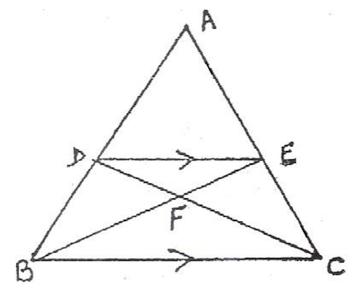
i. DE || BC and $\frac{AD}{DB} = \frac{3}{2}$

To find:

i. $\frac{AD}{AB}$

ii. $\frac{DE}{BC}$

iii. $\frac{A(\Delta DEF)}{A(\Delta DEC)}$



To Prove That:

i. Prove: ΔDEF ~ ΔCFB

ii. Prove: ΔADE ~ ΔABC

Statement	Reason
(i) (1) $\frac{AD}{DB} = \frac{3}{2}$; (2) $\frac{AD}{AB} = \frac{3}{3+2} = \frac{3}{5}$	Given BY BPT
(ii) (1) In ΔADE AND ΔABC (a) ∟A = ∟A (b) ∟ADE = ∟ABC (c) ΔADE ~ΔABC (2) $\frac{AD}{DB} = \frac{DE}{BC} = \frac{AE}{AC}$; $\frac{DE}{BC} = \frac{3}{5}$	 Common angle Corresponding angles By AA postulate By BPT and from statement 1
(iii) In ΔDEF and ΔCFB ; (a) ∟DFE = ∟BFC (b) ∟DEF = ∟FBC (c) ΔDEF ~ ΔFBC (2) $\frac{DF}{FC} = \frac{EF}{FB} = \frac{DE}{BC} = \frac{3}{5}$; $\frac{A(\Delta DEF)}{A(\Delta DEC)} = \frac{DF}{DC} = \frac{DF}{DF+FC}$; $\frac{3}{3+5} = \frac{3}{8}$	 vertically opposite angles are equal Interior alternate angles By AA postulate By BPT A common base and a common angle.

Answer 8.

(a) ∑fx = x + x + 2 + x + 4 + x + 6 + x + 8 = 5x + 20

∑f = 5

X = $\frac{\sum fx}{\sum f}$

11 = $\frac{5(x+4)}{5}$

x = 11 – 4 = 7 ;

First three (∑fx) = 7 + (7 + 2) + (7 + 4) = 7 + 9 + 11 = 27

∑f = 3

X = $\frac{\sum fx}{\sum f}$ = $\frac{27}{3}$ = 9

(b) 5x – 11y = 2x + 5y

5x – 2x = 5y + 11y

3x = 16y

$$\frac{x}{y} = \frac{16}{3}$$

$$x = \frac{16}{3}y$$

$$x = \frac{3x^2 + 2y^2}{3x^2 - 2y^2}$$

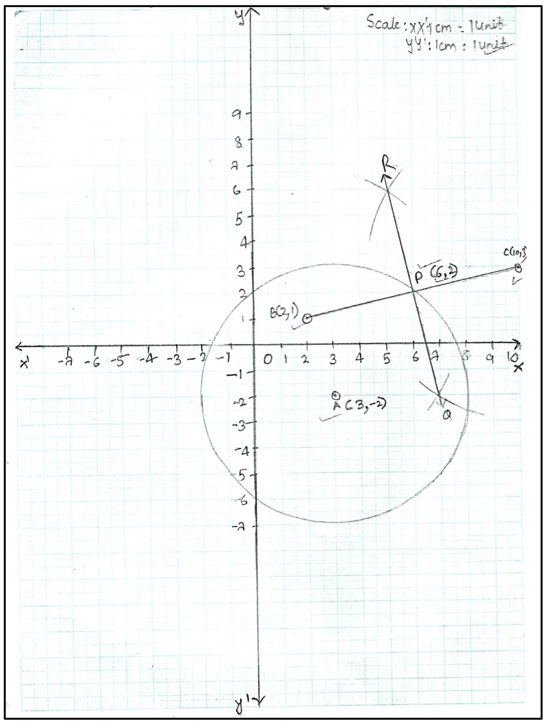
$$x = \frac{3\left(\frac{16}{3}y\right)^2 + 2y^2}{3\left(\frac{16}{3}y\right)^2 - 2y^2}$$

$$x = \frac{\frac{256}{3}y^2 + 2y^2}{\frac{256}{3}y^2 - 2y^2}$$

$$x = \frac{256\,y^2 + 6y^2}{256\,y^2 - 6y^2}$$

$$x = \frac{131}{125}$$

(c)



(i) Locus of point equidistant from B and C is anywhere on line RQ.

(ii) Circle on graph.

(iii) P is marked on the circumference on the graph.

(iv) Coordinates of P = (6, 2)

Answer 9.

(a) Multiples of 4 between 10 and 250 are

12, 16, 20, 24, ………, 248

Here, a = 12,

d = 4,

l = 248

$\therefore T_n = a + (n-1)\,d$

$248 = 12 + (n-1) \times 4$

$248 - 12 = 4(n-1)$

$236 = 4(n-1)$

$\frac{236}{4} = n - 1$

$n - 1 = 59$

$\therefore n = 59 + 1 = 60$

∴ Number of multiples of 4 are 60.

(b) In G.P.

$T_1 = -3$

$(T_2)^2 = T_4$, Find T_9

Let a be the first term and r be the common ratio, then

$T_1 = a = -3$

$T_4 = (T_2)^2$

$ar^3 = (ar)^2$

$ar^3 = a^2r^2$

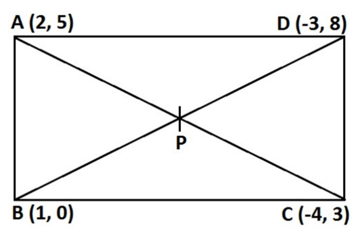
$r = a = -3$

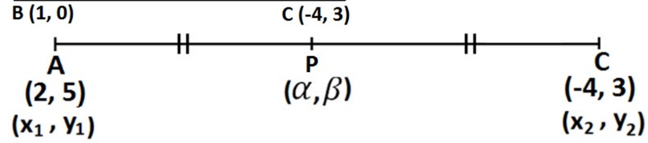
$\therefore T_7 = ar^6 = 3\,(-3)^6$

$= -3 \times 729$

$= \underline{\underline{-2187}}$

(c) i.





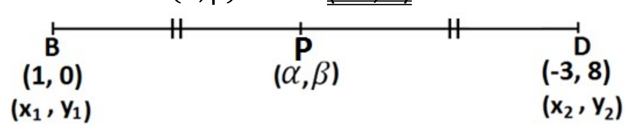
$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$\alpha = \frac{2-4}{2}$; $\beta = \frac{5+3}{2}$

$\alpha = \frac{-2}{2}$; $\beta = \frac{8}{2}$

$\alpha = -1$; $\beta = 4$

$P(\alpha, \beta) = \underline{\underline{(-1, 4)}}$



$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$= \frac{-1-3}{2}$; $= \frac{0+8}{2}$

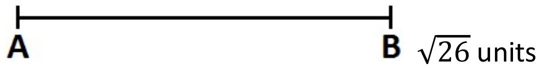
$= \frac{-2}{2}$; $= \frac{8}{2}$

$= -1$; $= 4$

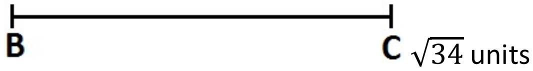
$Q(\alpha, \beta) = \underline{\underline{(-1, 4)}}$

ii.

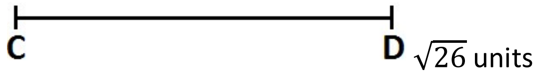
1. Side AB :



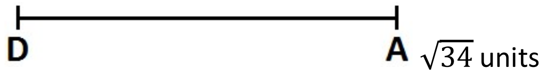
2. Side BC :



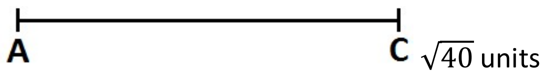
3. Side CD :



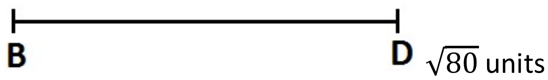
4. Side DA :



5. Diagonal AC :



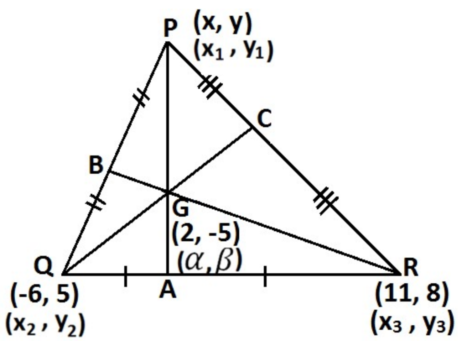
6. Diagonal BD :



Since the opoosite sides are equal and the diagonals are unequal so the quadrilateral ABCD is a parallelogram

Answer 10.

(a)



α	=	$\frac{x_1 + x_2 + x_3}{3}$	;	β	=	$\frac{y_1 + y_2 + y_3}{3}$
2	=	$\frac{x - 6 + 11}{3}$	;	-5	=	$\frac{y + 5 + 8}{3}$
			;	-5	=	$\frac{y + 13}{3}$
6	=	$x + 5$	;	$y + 13$	=	-15
x	=	$6 - 5$	;	y	=	$-15 - 13$
x	=	1	;	y	=	-28

P (x, y) = <u>(1, –28)</u>

(b) L.H.S

(sin A + cosec A)^2 + (cos A + sec A)^2

= $\sin^2 A + 2 \sin A \operatorname{cosec} A + \operatorname{cosec}^2 A + \cos^2 A + 2 \cos A \sec A + \sec^2 A$

= $\sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2 \sin A \times \frac{1}{\sin A} + 2 \cos A \times \frac{1}{\cos A}$

= $1 + \operatorname{cosec}^2 A + \sec^2 A + 2 + 2$

= $5 + \operatorname{cosec}^2 A + \sec^2 A$

= $5 + 1 + \cot^2 A + 1 + \tan^2 A$

= $7 + \tan^2 A + \cot^2 A$

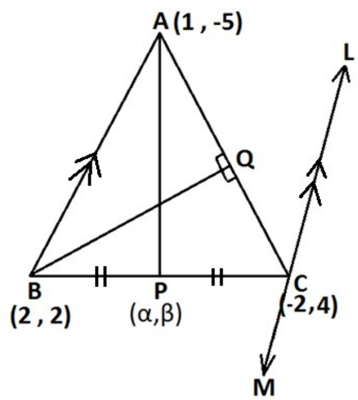
R.H.S

$7 \tan^2 A + \cot^2 A$

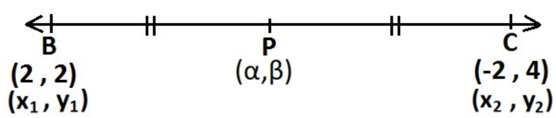
L.H.S = R.H.S

Hence proved

(c)



i.



$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$\alpha = \frac{(2) - (2)}{2}$; $\beta = \frac{(2) + (4)}{2}$

$\alpha = \frac{0}{2}$; $\beta = \frac{6}{2}$

$\alpha = 0$; $\beta = 3$

P (α, β) = (0, 3)

Equation of AP:

A = (1, -5) = x_1, y_1

P = (0, 3) = x_2, y_2

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{(y) - (-5)}{(x) - (1)} = \frac{(3) - (-5)}{(0) - (1)}$$

$$\frac{y+5}{x-1} = \frac{8}{-1}$$

$-y - 5 = 8x - 8$

$-8x - y = -8 + 5$

$-8x - y = -3$

$\underline{\underline{8x + y}} = \underline{\underline{3}}$

ii. m of AC:

A = $(1, -5) = (x_1, y_1)$

C = $(-2, 4) = (x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-5)}{-2-1} = \frac{9}{-3} = -3$$

$$m' \text{ of } BQ = \frac{-1}{m \text{ of } AC} = \frac{-1}{-3} = \frac{1}{3}$$

<u>Equation of BQ:</u>

$m = \frac{1}{3}$

B = $(2, 2) = x_1, y_1$

$$m = \frac{y - y_1}{x - x_1}$$

$$\frac{1}{3} = \frac{y-2}{x-2}$$

$x - 2 = 3(y - 2)$

$x - 2 = 3y - 6$

$\underline{\underline{x - 3y}} = \underline{\underline{-4}}$

iii. Slope of AB:

A = $(1, -5) = (x_1, y_1)$

B = $(2, 2) = (x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2)-(-5)}{(2)-(1)} = \frac{2+5}{1} = \frac{7}{1} = \underline{\underline{7}}$$

∴ m' of LM = m of AB, parallel = $\underline{\underline{7}}$

∴ <u>Equation of line LM</u>

$m = 7$

C = $(-2, 4) = (x_1, y_1)$

$$m = \frac{y - y_1}{x - x_1}$$

$$7 = \frac{(y) - (4)}{(x) - (-2)}$$

$$7 = \frac{y-4}{x+2}$$

$7(x + 2) = y - 4$

$7x + 14 = y - 4$

$\underline{\underline{7x - y}} = \underline{\underline{-18}}$

Answer 11.

(a) Number of total outcomes = {HH, HT, TT, TH}

= 4

i. Favourable outcomes = two heads

= {HH}

No.of favourable outcomes = 1

P (two heads) = $\frac{Favourable\ no.of\ outcomes}{Total\ number\ of\ outcomes}$ = $\frac{1}{4}$

ii. Favourable outcome = at least 1 head

= {HT, TT, TH}

Favourable no. of outcomes = 3

P(at least 1 head) = $\frac{Favourable\ no.of\ outcomes}{Total\ number\ of\ outcomes}$ = $\frac{3}{4}$;

iii. Favourable outcome = at most 1 head = {HT, TT, TH}

Number of favourable outcomes = 3

P (at most 1 head) = $\frac{Favourable\ no.of\ outcomes}{Total\ number\ of\ outcomes}$ = $\frac{3}{4}$

(b) f (x) = $x^2 + px + q$

g (x) = $x^2 + mx + n$

x + a = 0

x = $\underline{\underline{-a}}$

∴ f (– a) = 0

f (x) = $x^2 + px + q$

f (– a) = $(-a)^2 + p(-a) + q$

0 = $a^2 - ap + q$ ----- (1)

g (– a) = 0

g (x) = $x^2 + mx + n$

g (– a) = $(-a)^2 + m(-a) + n$

0 = $a^2 - ma + n$ ------ (2)

Since the factor is the same for both the equations, we get

$a^2 - ap + q = a^2 - ma + n$

$a^2 - ap - a^2 + ma = n - q$

$-ap + ma = n - q$

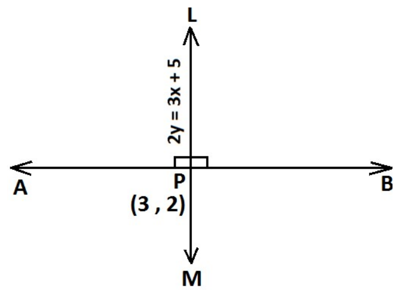
$a(-P + m) = n - q$

$a = \frac{n-q}{m-p}$

<u>Hence proved</u>

(c)

i.



m of line LM:

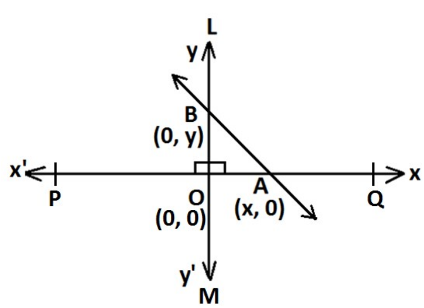
2y	=	3x + 5
y	=	$\frac{3}{2}$ x + $\frac{5}{2}$
m	=	$\frac{3}{2}$
∴ m' of AB	=	$\left(\frac{-1}{m\ of\ LM}\right)$, perpendicular
	=	$\left(\frac{-1}{3/2}\right)$
	=	$\frac{-2}{3}$

Equation of line AB:

m	=	$\frac{-2}{3}$
P	=	(3, 2) = (x_1, y_1)
m	=	$\frac{y - y_1}{x - x_1}$
$\frac{-2}{3}$	=	$\frac{(y) - (2)}{(x) - (3)}$
$\frac{-2}{3}$	=	$\frac{y - 2}{x - 3}$
– 2 (x – 3)	=	3 (y – 2)
– 2x + 6	=	3y – 6

2x + 3y = 12

ii.



Since line AB meets A & B on coordinate axes; it will satisfy the given equation.

For point A:

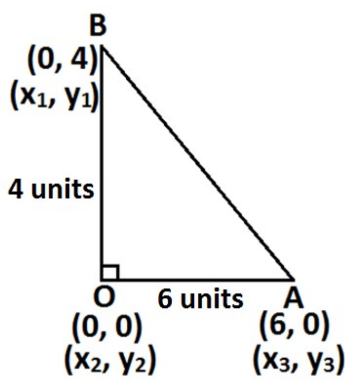
$2x + 3y$	=	12
$2x + 3(0)$	=	12
$2x$	=	12
x	=	6
A (x, 0)	=	(6, 0)

Since AB meets the y – axis at point B, abscissa of B is O.

For point B:

$2x + 3y$	=	12
$2(0) + 3y$	=	12
$3y$	=	12
y	=	$\frac{12}{3}$
Y	=	4
B (0, y)	=	(0, 4)

iii.



From the diagram,

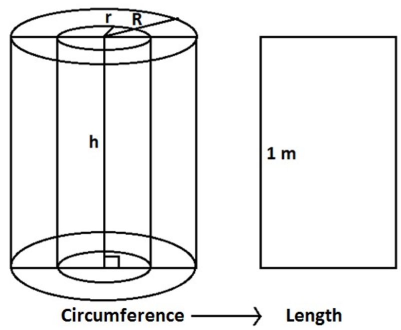
x-intercept	=	6
y-intercept	=	4
∴ Area of A	=	$\frac{1}{2} \times b \times h$
	=	$\frac{1}{2} \times 6 \times 4$
	=	12sq. Units

Answers of Practice Paper 7

Section I

Answer 1.

(a)



Diameter of the hollow closed cylinder = 20 cm

∴ Radius (r) $= \frac{20}{2} = 10$ cm

and height (h) = 35 cm

Total area $= 2\pi r(r + h)$ $= 2 \times \frac{22}{7} \times 10\,(10 + 35)\ \text{cm}^2$

$= \frac{440}{7} \times 45$ $= \frac{19800}{7}\ \text{cm}^2$

Width of sheet = 1 m = 100 cm

∴ Length of sheet required $= \frac{19800}{7 \times 100}$ $= \frac{198}{7}$ cm = 28 cm

Rate of single sheet = Rs. 56 per m

∴ Cost $= 28 \times \frac{56}{100}$ $= \frac{1568}{100}$ = <u>Rs. 15.68</u>

Area of sheet $= \frac{19800}{7}\ \text{cm}^2$

Total wastage in cutting, overlapping etc. $= \frac{19800}{7} \times \frac{10}{100}$ $= \frac{1980}{7}$ cm $= 10\ \%$ of $\frac{19800}{7}$

Total area of sheet required $= \frac{19800}{7} + \frac{1980}{7}$ $= \frac{21780}{7}\ \text{cm}^2$

$= 3111.4\ \text{cm}^2$ = <u>$3111\ \text{cm}^2$</u>

(b) R.H.S $= (\sec A - \tan A)^2$ $= \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2$ $= \frac{(1-\sin A)^2}{(\cos A)^2}$

$= \frac{(1-\sin A)^2}{1-\sin^2 A}$ $= \frac{(1-\sin A)(1-\sin A)}{(1+\sin A)(1-\sin A)}$ $= \frac{1-\sin A}{1+\sin A}$

L.H.S $= \frac{1-\sin A}{1+\sin A}$

<u>L.H.S = R.H.S</u>

<u>Hence Proved.</u>

(c) f(x) = $2x^3 + ax^2 - 11x + b$

When divided by (x – 2)

f(2)	=	0 ; given
f(2)	=	$2(2)^3 + (a)(2)^2 - 11(2) + b$
0	=	2(8) + (a)[4] – 22 + b
6	=	4a + b............................... (i)

When divided by (x – 3)

f[3]	=	42 ; given
f[3]	=	$2[3]^3 + (a)[3]^2 - 11[3] + b$
42	=	2(27) + (a)(9) – 33 + b
9a + b	=	21............................... (ii)

Subtracting Equation (i) and (ii)

we get a	=	3
Substituting a	=	3 in eq. (i)
we get b	=	–6

Now substituting a and b

f(x) = $2x^3 + 3x^2 - 11x - 6$ dividing it by x – 2

Quotient = $2x^2 + 7x + 3$ = (2x + 1) (x + 3)

$2x^3 + 3x^2 - 11x - 6$ = (x – 2) (2x + 1) (x + 3)

Answer 2.

(a)

$$\sqrt{3x+1} - \sqrt{x-1} = 2$$

$$\sqrt{3x+1} = 2 + \sqrt{x-1}$$; Squaring both sides

$$(\sqrt{3x+1})^2 = (2 + \sqrt{x-1})^2$$

$$3x + 1 = 4 + 4\sqrt{x-1} + x - 1$$

$$3x + 1 = 3 + x + 4\sqrt{x-1}$$

$$(x - 1) = 2\sqrt{x-1}$$; square both sides

$$x^2 - 2x + 1 = 4x - 4$$

$$x^2 - 6x + 5 = 0$$

$$(x - 5)(x - 1) = 0$$

x = 5 or x = 1

x = {5, 1}

(b)

$$x = \frac{8ab}{a+b}$$

$$\frac{x}{4a} = \frac{2b}{a+b}$$

By Componendo and Dividendo;

$\frac{x+4a}{x-4a} = \frac{2b+a+b}{2b-a-b}$

$\frac{x+4a}{x-4a} = \frac{3b+a}{b-a}$ .... (i)

x $= \frac{8ab}{a+b}$

$\frac{x}{4b} = \frac{2a}{a+b}$

By componendo and dividendo;

$\frac{x+4b}{x-4b} = \frac{2b+a+b}{2b-a-b}$

$\frac{x+4b}{x-4b} = \frac{3a+b}{a-b}$ .... (ii)

$\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$

Subs eq. (i) and (ii)

$\frac{3b+a}{b-a} + \frac{3a+b}{a-b} = \frac{3b+a-(3a+b)}{(b-a)} = \frac{2(b-a)}{(b-a)} = \underline{\underline{2}}$

(c) In ΔABC

tan $30^0 = \frac{AB}{BC}$

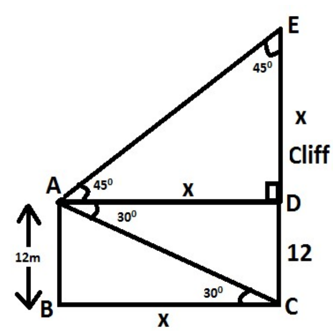
$\frac{1}{\sqrt{3}} = \frac{12}{BC}$

BC $= 12\sqrt{3}\ cm$

$12 \times 1.732 = 20.784$m

tan $45^0 = \frac{ED}{AD} = \frac{ED}{12\sqrt{3}}$;

ED $= 12\sqrt{3}$



Total ht. of cliff = AB + ED

$= 12 + 12\sqrt{3} =$ <u>32.784 m</u>

Answer 3.

(a)

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} 2+0 \\ 0-3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

By equalizing matrices: x = 2, y = –3

(b) L.H.S: $\frac{1 + cos\alpha}{1 - cos\alpha}$

$= \frac{(1 + cos\alpha)(1 + cos\alpha)}{(1 - cos\alpha)(1 + cos\alpha)} = \frac{(1 + cos\alpha)^2}{(1 - cos^2\alpha)} = \frac{(1 + cos\alpha)^2}{sin^2\alpha}$

$= \left(\frac{1}{\sin\alpha} + \frac{cos\alpha}{sin\alpha}\right)^2 = (\text{cosec } \alpha + \cot \alpha)^2$

L.H.S = R.H.S

(c) Let no. of arrows Arjun had be 'x';

$\frac{x}{2}$ = arrows used cut down

Balance = $x - \frac{x}{2} = \frac{x}{2}$

Rath driver = 6

rath = 1

flag = 1

bow = 1

Total = 9............................... (i)

Bheeshama = $1 + 4\sqrt{x}$;

Total = $\frac{x}{2} = 10 + 4\sqrt{x}$

$x = 20 + 8\sqrt{x}$

Square both sides;

$(8\sqrt{x})^2 = (20 - x)^2$

$64x = 400 + x^2 - 40x$

$0 = x^2 - 104x + 400$

$0 = x^2 - 100x - 4x + 400$

$0 = (x - 100)$ or $(x - 4)$

Since arrows are more than 4 from (i) x = 100

Answer 4.

(a)

$A = \{x : 4<3x - 2 \leq 13; x \in R\}$

$4<3x - 2$; $3x - 2 \leq 13$

$4 + 2<3x$; $3x \leq 15$

$2<x$; $x \leq 5$

$2 < x \leq 5$

$B = \{x : -2 \leq 5 + 7x < 40 ; x \in R\}$

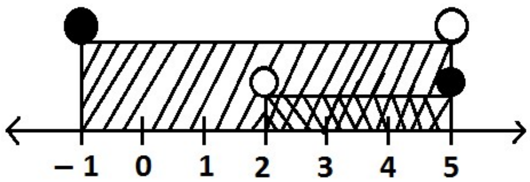
$-2 \leq 5 + 7x$; $5 + 7x < 40$

$-7 \leq 7x$; $x < 5$

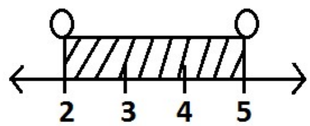
-1 < x ; x < 5

-1 ≤ x < 5

A ∩ B = { x : x ; 2 < x < 5 ; x ∈ R }



OR



(b) Number of total outcomes = 48

i. Favourable outcomes = number divisible by 7

= {7, 14, 21, 28, 35, 42, 49}

Number of favourable outcomes = 7

P (number divisible by 7) = $\frac{Favourable\ no.of\ outcomes}{Total\ number\ of\ outcomes}$ = $\frac{7}{48}$

ii. Favourable outcome = a prime number less than 25

= {3, 7, 11, 13, 17, 19, 23}

Favourable no. of outcomes = 7

P (prime number less than 25) = $\frac{Favourable\ no.of\ outcomes}{Total\ number\ of\ outcomes}$ = $\frac{7}{48}$

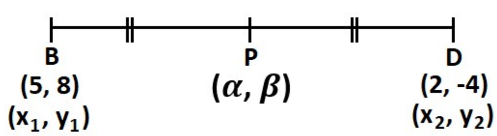
iii. Favourable outcome = a number which is a perfect square

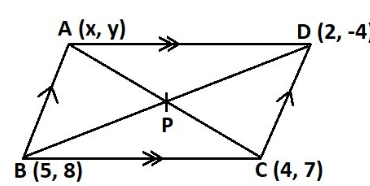
= {4, 9, 16, 25, 49}

Number of favourable outcomes = 6

P (a number which is a perfect square) = $\frac{Favourable\ no.of\ outcomes}{Total\ number\ of\ outcomes}$ = $\frac{6}{48}$

(c)





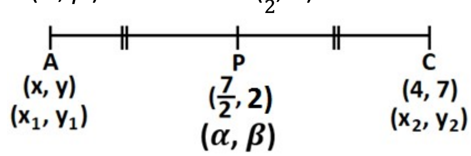
$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$= \frac{5+2}{2}$; $= \frac{8-4}{2}$

$= \frac{7}{2}$; $= \frac{4}{2}$

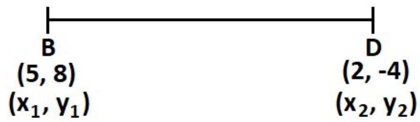
$= 2$

P (α, β) = $(\frac{7}{2}, 2)$



α	$= \frac{x_1 + x_2}{2}$	;	β	$= \frac{y_1 + y_2}{2}$
$\frac{7}{2}$	$= \frac{x + 4}{2}$	;	2	$= \frac{y + 7}{2}$
7	$= \frac{7}{2}$	;	4	= y + 7
x	= 3	;	y	= – 3

A (x, y) = (3, – 3)



$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

$\frac{-4 - 8}{2 - 5} = \frac{y - 8}{x - 5}$

– 12 (x – 5) = – 3 (y – 8)

– 12 x + 60 = – 3y + 24

– 12x + 3y = – 36

12x – 3y = 36

4x – y = 12

Section II

Answer 5.

(a) Volume of remaining solid $= \pi r^2 H - \frac{1}{3}\pi r^2 h$

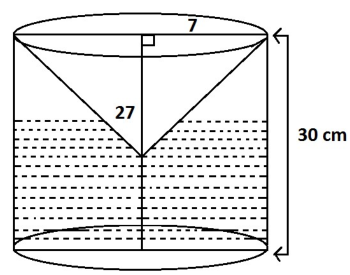
$= \pi (7^2 \times 30 - \frac{1}{3} \times 7^2 \times 24)$

$= \frac{22}{7} \times 7^2 (30 - \frac{24}{3})$

$= 22 \times 7 (30 - 8)$

$= 22 \times 7 (22)$

$= 3388 \text{ cm}^3$



$l = \sqrt{27^2 + 7^2} = \sqrt{729 + 49}$

$= \sqrt{778} = 27.89$

TSA of remaining solid = CSA of cylinder + CSA of cone + bottom circular area

$= 2\pi r H + \pi r l + \pi r^2$

$= \pi r (2H + l + r)$

$= \frac{22}{7} \times 7 (2 \times 30 + \sqrt{680} + 7) = 22 \times (60 + \sqrt{680} + 7)$

$= 22 \times (67 + 27.89) = 22 \times 94.89$

= 2087.58 sq.cm

(b) $\frac{a^3 + 3ab^2}{b^3 + 3a^2 b} = \frac{63}{62}$

By componendo and dividendo

$$\frac{a^3 + 3ab^2 + b^3 + 3a^2b}{a^3 + 3a^2b - a^3 - 3ab^2} = \frac{63 + 62}{63 - 62}$$

$$\frac{(a + b)^3}{(a - b)^3} = \frac{125}{1}$$

Cube roots both sides

$$\frac{a + b}{a - b} = \frac{5}{1}$$

By componendo and dividendo

$$\frac{a + b + (a - b)}{a + b - (a - b)} = \frac{5 + 1}{5 - 1}$$

$$\therefore \frac{a}{b} = \frac{3}{2}$$

(c) P = (0, 3a) = (x_1, y_1)

Q = (2a, 0) = (x_2, y_2)

$$\alpha = \frac{x_1 n + x_2 m}{m + n}$$

$$1 = \frac{3 \times 0 + x \times 2}{5}$$

$$x = \frac{5}{2}$$

$$\beta = \frac{y_1 n + y_2 m}{m + n}$$

$$1 = \frac{3y + 2 \times 0}{5}$$

$$y = \frac{10}{3}$$

P = x, y = $\left(0, \frac{10}{3}\right)$

Q = x, y = $\left(\frac{5}{2}, 0\right)$

Equation of line PQ: P = x, y = $\left(0, \frac{10}{3}\right) = (x_1, y_1)$;

Q = x, y = $\left(\frac{5}{2}, 0\right)$ = (x_2, y_2)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\frac{0 - \frac{10}{3}}{\frac{5}{2} - 0} = \frac{y - \frac{10}{3}}{x - 0}$$

$$\frac{-\frac{10}{3}}{\frac{5}{2}} = \frac{y - \frac{10}{3}}{x}$$

$\therefore$ 50 = 20x + 15y

10 = 4x + 3y

Answer 6.

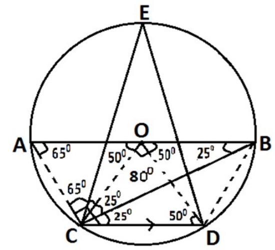
(a) Given:

$\angle ABC = 25°$

To find:

$\angle AOC$

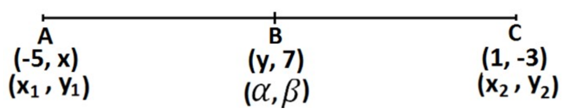
$\angle BOD$



Construction: Join AC, CO, BD

Statement	Reason
1) $\angle AOC = 2\ \angle ABC$ $= 2 \times 25 = \underline{\underline{50°}}$	Angle subtended at the centre is double that of the circumference.
2) In ΔABC, $\angle ACB = 90°$ $\angle CAB = 180 - (90 + 25)$ $= 180 - 115 = 65°$	Angle in a semicircle is a right angle. Sum of all angles of triangle is 180.
3) $\angle OCD = \angle AOC = 50°$	Alternate interior angles.
4) $\angle BOD = \angle OCD = \underline{\underline{50°}}$	Exterior angle is equal to interior angle.

(b)



$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$y = \frac{-5 + 1}{2}$; $7 = \frac{x - 3}{2}$

$y = \frac{-4}{2}$; $14 = x - 3$

$y = \underline{\underline{-2}}$; $x = 14 + 3$

$x = \underline{\underline{17}}$

(c) Fv = Rs. 50

n = ?

r = 10%

mv = ?

1 = x

D = 750

R = 8%

Let n = 1

$D = \frac{Fvnr}{100} = \frac{50 \times 1 \times 10}{100}$ = Rs.5

100, 8, ?, 500

?	=	$\frac{500}{8}$ = 62.50
mv	=	Rs.62.50
D	=	$\frac{Fvnr}{100}$
750	=	$\frac{50 \times n \times 10}{100}$
n	=	150 shares
I	=	mv × n = 62.5 × 150 = Rs. 9375

Answer 7.

(a)

n	=	?
x	=	Rs. 1200
r	=	8 %
MV	=	Rs. 12440
MV	=	n x + $\left[\frac{n\,x\,r\,(n+1)}{2400}\right]$
12440	=	n × 1200 + $\left[\frac{n \times 1200 \times 8 \times n + 1}{2400}\right]$
12440	=	1200n + 4n (n + 1)
12440	=	$1200n + 4n^2 + 4n$
0	=	$4n^2 + 1204n - 12440$
0	=	$n^2 + 301n - 3110$
0	=	$n^2 + 311n - 10n - 3110$
0	=	n (n + 311) – 10 (n + 311)
0	=	(n + 311) (n – 10)
n + 311= 0	or	n – 10 = 0
n	=	– 311 or n = 10

Ignoring – ve,

n = 10 months

(b) Let n – 1, n + 1, be factors → $x \neq 0$

Let x – 2 be the factor

x – 2 = 0

∴ x = 2

f(x) = $3x^3 + 2x^2 - 19x + 6$

f(2) = $3(2)^3 + 2(2)^2 - 19(2) + 6$

f(2) = 24 + 8 – 36 + 6

f(2) = 0

∴ (x – 2) is a factor of f(x)

$$
\begin{array}{r|l}
 & 3x^2 + 8x - 3 \\
\hline
x - 2 & 3x^3 + 2x^2 - 19x + 6 \\
 & 3x^3 - 6x^2 \\
 & (-) \quad (+) \\
\hline
 & 8x^2 - 19x \\
 & 8x^2 - 16x \\
 & (-) \quad (+) \\
\hline
 & -3x + 6 \\
 & -3x + 6 \\
 & (+) \quad (-) \\
\hline
 & 0 \\
\hline
\end{array}
$$

f(x) = $(3x^2 + 8x - 3)(x-2)$

= $(x - 2)(3x^2 - x + 9x - 3)$

= $(x - 2)[x(3x - 1) + 3(3x - 1)]$

= $\underline{(x - 2)(x + 3)(3x - 1)}$

(c) In an A.P.

$T_4 + T_8$ = 24

$T_6 + T_{10}$ = 44

Let a be the first term and d be the common difference

∴ a + 3d + a + 7d = 24

2a + 10d = 24

a + 5d = 12 .... (i)

a + 5d + a + 9d = 44

2a + 14d = 44

a + 7d = 22 .... (ii)

Subtracting (i) from (ii)

2d = 10

d = $\frac{10}{2}$

d = 5

a + 5d = 12

a + 5 × 5 = 12

a + 25 = 12

a = 12 – 25 = –13

∴ Three terms of A.P. will be –13, –8, –3

Answer 8.

(a) a = – 3

$(t_2)^2 = t_4$

$(ar)^2 = (ar^3)$

$(-3r)^2 \quad = (-3r^3)$

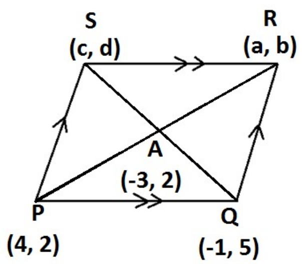
$9r^2 \quad = -3r^3$

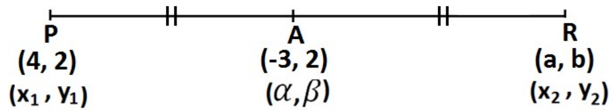
$\underline{\underline{-3}} \quad = r$

$t_7 \quad = ar^{n-1} \quad = ar^{7-1} \quad = (-3) \times (-3)^6$

$\quad = -3 \times 729 \quad = \underline{\underline{-2187}}$

(b)





$\alpha = \frac{x_1 + x_2}{2} \quad ; \quad \beta = \frac{y_1 + y_2}{2}$

$-3 = \frac{4 + a}{2} \quad ; \quad 2 = \frac{2 + b}{2}$

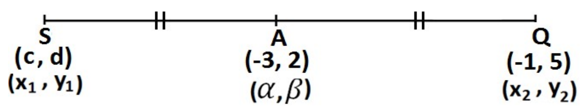
$-6 = 4 + a \quad ; \quad 4 = 2 + b$

$-a = 4 + 6 \quad ; \quad b = 4 - 2$

$-a = 10 \quad ; \quad b = 2$

$a = -10$

$R\ (a, b) = \underline{\underline{(-10, 2)}}$



$\alpha = \frac{x_1 + x_2}{2} \quad ; \quad \beta = \frac{y_1 + y_2}{2}$

$-3 = \frac{c - 1}{2} \quad ; \quad 2 = \frac{d + 5}{2}$

$-6 = c - 1 \quad ; \quad 4 = d + 5$

$C = -6 + 1 \quad ; \quad d = 4 - 5$

$C = -5 \quad ; \quad d = -1$

$S\ (c, d) = \underline{\underline{(-5, -1)}}$

(c) Y – axis.

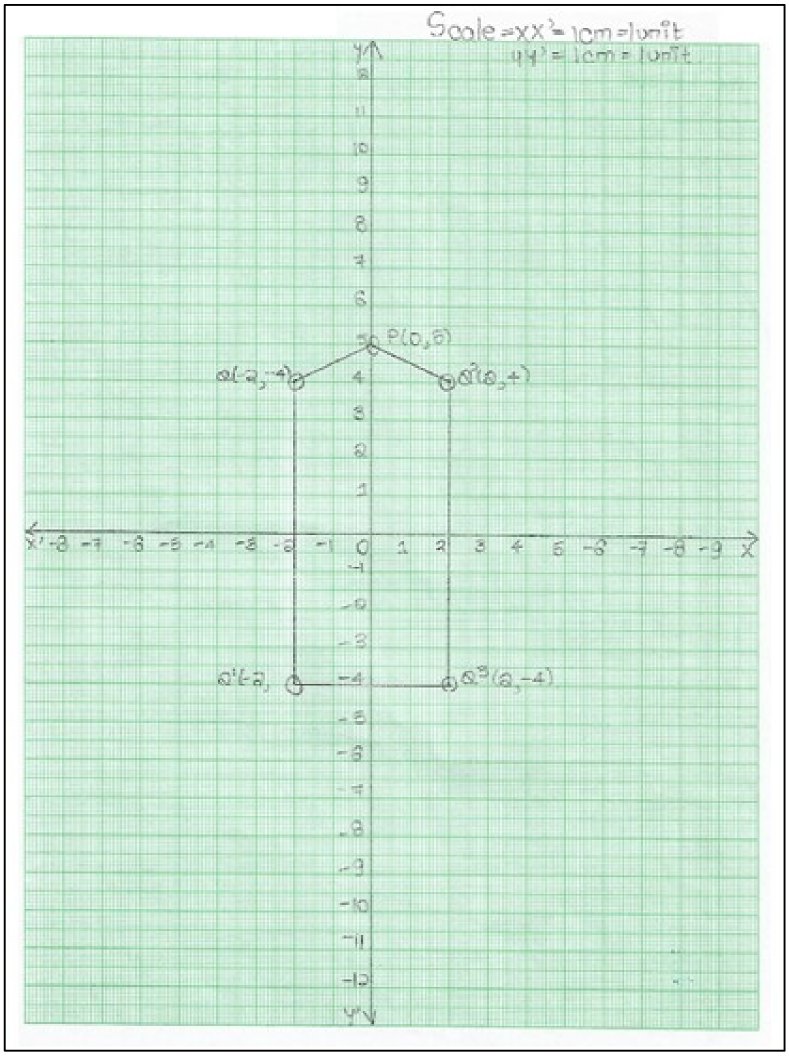
i. $Q\ (2, -4) \xrightarrow{Mx} Q^1\ (-2, -4)$

$Q'(-2, 4) \xrightarrow{My} Q^2\ (2, 4)$

$Q'\ (-2, 4) \xrightarrow{Mo} Q^3\ (2, -4)$

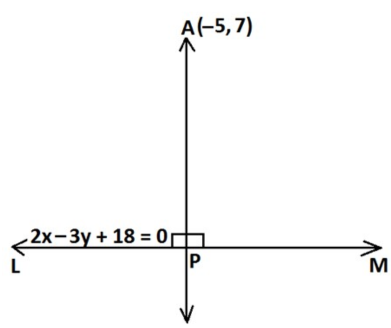
ii. Irregular Pentagon.

iii. $x = 0$



Answer 9.

(a) i.



For line LM

$2x - 3y + 18$	=	0 .... (i)
y	=	mx + c
– 3y	=	– 2x – 18
y	=	$\left(\frac{-2}{-3}\,x\right) + \left(\frac{-18}{-3}\right) \;=\; \frac{2}{3}x + 6$
m	=	$\frac{2}{3}$

m′ of AP = $\left(\frac{1}{m' \, of \, LM}\right)$, perpendicular = $\left(\frac{1}{2/3}\right)$ = $\left(\frac{-3}{2}\right)$

For line AP

m = $\frac{-3}{2}$

A (x_1, y_1) = $(-5, 7)$

m = $\frac{y - y_1}{x - x_1}$

$\frac{-3}{2} = \frac{(y) - (7)}{(x) - (-5)}$

$\frac{-3}{2} = \frac{y - 7}{x + 5}$

$-3(x + 5) = 2(y - 7)$

$-3x - 15 = 2y - 14$

$-3x - 2y = -14 + 15$

$-3x - 2y = 1$

Or

$3x + 2y + 1 = 0$ .... (ii)

ii. ∴ For finding P, we equal (i) and (ii)

$2x - 3y = -18 \quad (\times 2)$

$3x + 2y = -1 \quad (\times 3)$

$4x - 6y = -36$

$9x + 6y = -3$

$13x = -39$

$x = \frac{-39}{13} = -3$

∴ On substituting the value of x in equation (i) we get

$3x + 2y = -1$

$3(-3) + 2y = -1$

$-9 + 2y = -1$

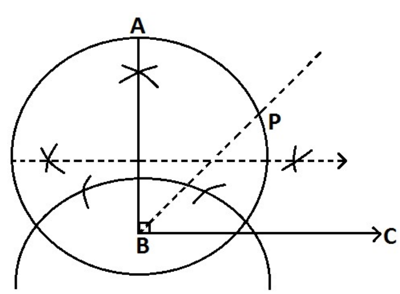
$2y = -1 + 9$

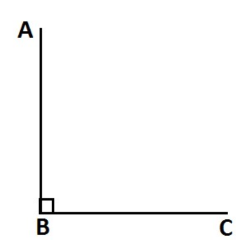
$2y = 8$

$y = \frac{8}{2} = 4$

P (x, y) = (– 3, 4)

(b)





PB = __________

(c)

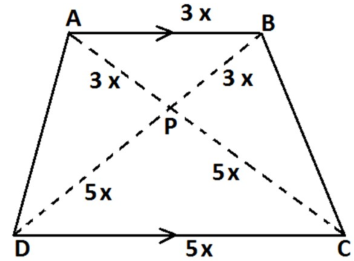
Given:

i. AB || DC ii. AP : CP = 3 : 5

To find:

i. $\frac{A\,\Delta APB}{A\,\Delta CPB}$ ii. $\frac{A\,\Delta DPC}{A\,\Delta APB}$

iii. $\frac{A\,\Delta ADP}{A\,\Delta APB}$ iv. $\frac{A\,\Delta APB}{A\,\Delta ADB}$



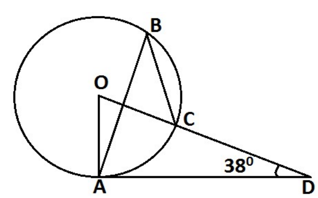
	Statement	Reason
1.	In Δ APB and ΔCPD, ∠APB = ∠CPD ∠ABP = ∠PDC Δ APB ∼ Δ DPC	 Vertically opposite angles. Interior alternate angles. By A. A. postulate.
2.	$\frac{AP}{CP} = \frac{PB}{PD} = \frac{AB}{CD} = \frac{3}{5}$	By B.P.T and given.
3.	$\frac{A\,\Delta\,APB}{A\,\Delta\,CPB} = \frac{AP}{PC} = \frac{3}{5} = \underline{\underline{3:5}}$	If many triangles have the common vertex and bases are along the same line, the ratio between their areas is equal to the ratio between the length of their bases.
4.	$\frac{A\,\Delta\,DPC}{A\,\Delta\,APB} = \frac{(PC)^2}{(PA)^2}$ $= \frac{(5x)^2}{(3x)^2} = \frac{25x^2}{9x^2}$ $= \frac{25}{9} = \underline{\underline{25:9}}$	Areas of two similar triangles are proportional to the squares of their corresponding sides.
5.	$\frac{A\,\Delta\,ADP}{A\,\Delta\,APB} = \frac{DP}{PB}$ $= \frac{5}{3} = \underline{\underline{5:3}}$	If many triangles have a common vertex and their bases lie along the same straight line, then the ratio between their areas is equal to the ratio between the lengths of their bases.
6.	$\frac{A\,\Delta\,APB}{A\,\Delta\,ADB} = \frac{PB}{DB}$ $= \frac{3}{5+3}$ $= \frac{3}{8} = \underline{\underline{3:8}}$	If many triangles have the common vertex and their bases lie along the same straight line, then ratio between their areas is equal to the ratio between the lengths of their bases.

Answer 10.

(a)

Given: ∠CDA = 38°

To Find: ∠AOD; ∠ABC



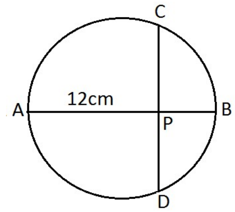
	Statement	Reason
1.	∠OAD = 90°	Radius ⊥ to tangent.
2.	∠AOD = 180° – (∠OAD + ∠ODA) = 180° – (90 + 38) = $\underline{\underline{52°}}$	Sum of all ∠s of a triangle is 180°.
3.	$∠ABC = \frac{1}{2}∠AOD$ $= \frac{1}{2} \times 52 = \underline{\underline{26°}}$	∠ subtended at the centre is double than that at the circumference.

(b) Given: i. PA = 12 cm

ii. PA = 2 PB

iii. PC = PD = x

To Find: 1. X



Statement	Reason
1) PA = 2 PB 12 = 2 PB PB = $\frac{12}{2}$ = 6 cm	Given On solving
2) PA × PB = PC × PD 12 × 6 = x × x x^2 = 72 x = $\sqrt{72} = 6\sqrt{2}$ cm = 8.484 cm	When 2 chords intersect internally then product of segment is equal

(c)

(x) marks	No. of Students	Cf
5	8	8
7	4	12
9	7	19
10	3	22
12	9	31
15	7	38
17	4	42
19	2	44

n = 44, even

$$Me = \frac{\left(\frac{n}{2}\right)^{th}\text{ term} + \left(\frac{n}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \frac{\left(\frac{44}{2}\right)^{th}\text{ term} + \left(\frac{44}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \frac{22^{nd}\text{ term} + 23^{rd}\text{ term}}{2}$$

$$= \frac{10+12}{2} = \frac{22}{2}$$

Me = 11 marks

Modal class = 12 marks

Answer 11.

(a)

Ht.(cm)	No. of students	Cf
140 – 145	12	12
145 – 150	20	32
150 – 155	30	62
155 – 160	38	100
160 – 165	24	124
165 – 170	16	140
170 – 175	12	152
175 – 180	8	160

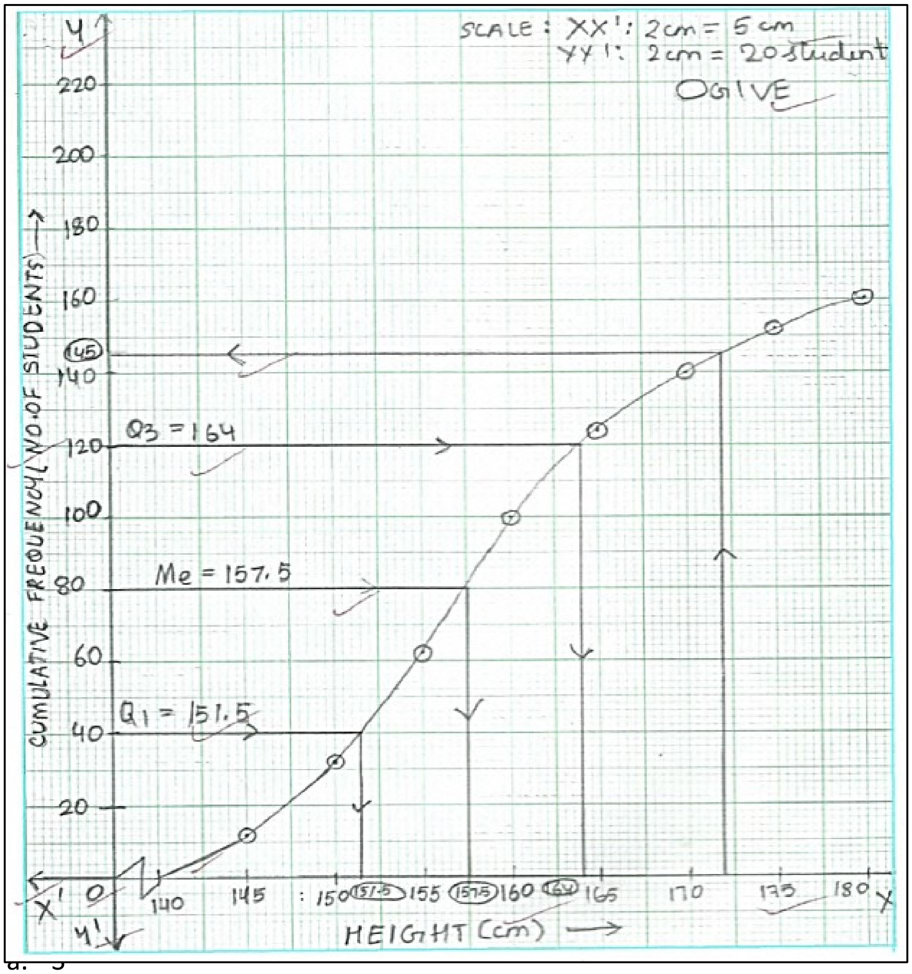
i. Median height = $\left(\frac{n}{2}\right)^{th}$ term = 80^{th} term = 157.5cm

ii. Upper Quartile Q_3 = $\left(\frac{3n}{4}\right)^{th}$ term = 120^{th} term = 164cm

Lower Quartile Q_1 = 40^{th} term = 151.5cm

Inter quartile range = $Q_3 - Q_1$ = 164 – 151.5 = 12.5cm

iii. No. of students whose ht is above 172cm = 160 – 145 = <u>15 students</u>



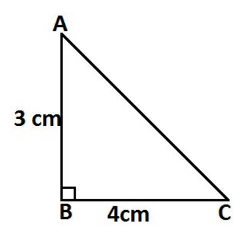
(b) Scale of map drawn of a triangular plot = 1 : ,50,000

Measurement of plot

AB = 3 cm,

BC = 4 cm and

∠ ABC = 90°



i. Actual length of AB = 3×250000 cm

$= \frac{3 \times 250000}{100}$ m $= \frac{3 \times 250000}{100 \times 1000}$ km

$= \frac{15}{2}$ = 7.5 km

And actual length of BC $= \frac{4 \times 250000}{100 \times 1000}$ = <u>10 km</u>

ii. Area of plot $= \frac{1}{2} \times BC \times AB$ $= \frac{1}{2} \times 7.5 \times 10$ km^2

$= \frac{75}{2}$ = <u>37.5 km^2</u>

Answers of Practice Paper 8

Section I

Answer 1.

(a) $\frac{a}{b} = \frac{b}{c} = k$

$\frac{a}{b} = k$

a = bk

$\frac{b}{c} = k$

b = ck

a = bk

a = ck. k

a = ck^2

L. H. S

$= \frac{a^2 + ab + b^2}{b^2 + bc + c^2}$ $= \frac{(ck^2)^2 + (ck^2)(ck) + (ck)^2}{(ck)^2 + ck(c) + c^2}$

$= \frac{c^2k^4 + c^2k^3 + c^2k^2}{c^2k^2 + c^2k + c^2}$ $= \frac{c^2k^2\ (k^2 + k + 1)}{-c^2\ (k^2 + k + 1)}$ $= \underline{\underline{k^2}}$

R. H. S

$= \frac{a}{c}$ $= \frac{ck^2}{c}$ $= \underline{\underline{k^2}}$

L.H.S = R. H. S

Hence proved.

(b) -2 < 2x – 6 or – 2x + 5 $\geq$ 13; R

-2 < 2x - 6 or -2x + 5 $\geq$ 13

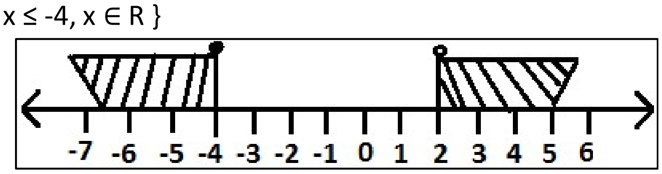
4 < 2x or -2x $\geq$ 8

2 < x or -4 $\geq$ x

∴ SS (x) = { x : x > 2 ; x ∈ R }

OR

SS (x) = { x : x ≤ -4, x ∈ R }



∴ SS = {ϕ} = null set

(c) n = 3 × 12 = 36 months

x = Rs. 200

MV = Rs. 8088

MV = nx + $\frac{nxr\ (n+1)}{2400}$

8088 = 7200 + $\frac{7200 \times r \times 37}{2400}$

$$\frac{888}{37 \times 3} = r$$
$$r = \underline{\underline{8\ \%}}$$

Answer 2.

(a) $(3m + 1)\, x^2 + 2\,(m + 1)\, x + m = 0$

a = $3m + 1$

b = $2m + 2$

c = m

D = $b^2 - 4ac = 0$

Since it has real and equal roots,

$0 = b^2 - 4ac$

$0 = (2m + 2)^2 - 4\,(3m + 1)\,(m)$

$0 = 4m^2 + 8m + 4 - 4m\,(3m + 1)$

$0 = 4m^2 + 8m + 4 - 12m^2 - 4m$

$0 = -8m^2 + 4m + 4$

$8m^2 - 4m - 4 = 0$

$2m^2 - m - 1 = 0$

$2m^2 - 2m + m - 1 = 0$

$2m\,(m - 1) + 1\,(m - 1) = 0$

$(2m + 1)\,(m - 1) = 0$

$2m + 1 = 0$ or $m - 1 = 0$

$m = -\frac{1}{2}$ or $m = 1$

$\therefore m = (-\frac{1}{2}, 1)$

(b) Let $\frac{a}{b} = \frac{c}{d} = k$; $a = bk$; $c = dk$

L.H.S

$$\frac{a^3\,c + ac^3}{b^3 d + bd^3} = \frac{(bk)^3\,dk + bk\,(dk)^3}{bd\,(b^2 + d^2)}$$
$$= \frac{b^3k^3\,dk + bkd^3k^3}{bd\,(b^2 + d^2)}$$
$$= \frac{bdk^4\,(b^2 + d^2)}{bd\,(b^2 + d^2)}$$
$$= k^4$$

RHS,

$$\frac{(a + c)^4}{(b + d)^4} = \frac{(bk + dk)^4}{(b + d)^4}$$
$$= \frac{k^4\,(b + d)^4}{(b + d)^4}$$
$$= k^4$$

$\therefore$ LHS = RHS

<u>Hence proved.</u>

(c) $f(x) = (a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$

$3x + 5 = 0$ (given it is a factor)

$3x = -5$

$x = \frac{-5}{3}$

$f\left(\frac{-5}{3}\right) = 0$

Using remainder Theorem,

$f(x) = (a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$

$f\left(\frac{-5}{3}\right) = (a - 1)\left(\frac{-5}{3}\right)^3 + (a + 1)\left(\frac{-5}{3}\right)^2 - (2a + 1)\left(\frac{-5}{3}\right) - 15$

$0 = (a - 1)\left(\frac{-125}{27}\right) + (a + 1)\left(\frac{25}{9}\right) - (2a + 1)\left(\frac{-5}{3}\right) - 15$

$0 = \frac{-125a}{27} + \frac{125}{27} + \frac{25a}{9} + \frac{25}{9} + \frac{10\,a}{3} + \frac{5}{3} - 15$

$\frac{15}{1} - \frac{5}{3} - \frac{25}{9} - \frac{125}{27} = \frac{-125a}{27} + \frac{25\,a}{9} + \frac{10a}{3}$

$\frac{405 - 45 - 75 - 125}{27} = \frac{-125a + 75a + 90a}{27}$

$160 = 40\,a$

$a = \frac{160}{40} = \underline{\underline{4}}$

On substituting the value of a in f (x) we get,

$f(x) = (4 - 1)x^3 + (4 + 1)x^2 - (2a + 1)x - 15$

$= 3x^3 + 5x^2 - [2(4) + 1]x - 15$

$= 3x^3 + 5x^2 - (8 + 1)x - 15$

$= 3x^3 + 5x^2 - 9x - 15$

$$\begin{array}{r|l} & x^2 - 3 \\ \hline 3x + 5 & 3x^3 + 5x^2 - 9x - 15 \\ & 3x^3 + 5x^2 \\ & (-)\quad (-) \\ \hline & \qquad\quad -9x - 15 \\ & \qquad\quad -9x - 15 \\ & \qquad\quad (+)\quad (+) \\ \hline & \qquad\qquad\quad 0 \end{array}$$

$3x^3 + 5x^2 - 9x - 15 = (3x + 5)(x^2 - 3)$

$= (3x + 5)[(x)^2 - (\sqrt{3})^2] = \underline{(3x + 5)(x + \sqrt{3})(x - \sqrt{3})}$

Answer 3.

(a) $\frac{(\sqrt{x+5} + \sqrt{x-16}) + (\sqrt{x+5} - \sqrt{x-16})}{(\sqrt{x+5} + \sqrt{x-16}) - (\sqrt{x+5} - \sqrt{x-16})} = \frac{7+3}{7-3}$ (By C & D)

$\therefore \frac{\sqrt{x+5} + \sqrt{x-16} + \sqrt{x+5} - \sqrt{x-16}}{\sqrt{x+5} + \sqrt{x-16} - \sqrt{x+5} + \sqrt{x-16}} = \frac{10}{4}$

$\frac{2\sqrt{x+5}}{2\sqrt{x-16}} = \frac{5}{2}$

$\frac{x+5}{x-16} = \frac{25}{4}$

$4x + 20 = 25x - 400$

$420 = 21x$

$x = \underline{\underline{20}}$

(b)

If (x – 2) is a factor,

x – 2 = 0 , ∴ x = 2

$f(x) = 2x^3 + 5x^2 - 11x - 14$

$f(2) = 2(2)^3 + 5(2)^2 - 11(2) - 14$

$f(2) = 16 + 20 - 22 - 14$

$f(2) = 36 - 36$

$f(2) = 0$

Since the remainder is zero,

(x – 2) is a factor.

$$
\begin{array}{r|l}
 & 2x^2 + 9x + 7 \\
\hline
x-2 & 2x^3 + 5x^2 - 11x - 14 \\
 & {}_{(-)}2x^3 - {}_{(+)}4x^2 \downarrow \\
\hline
 & 9x^2 - 11x \\
 & {}_{(-)}9x^2 - {}_{(+)}18x \\
\hline
 & 7x - 14 \\
 & {}_{(-)}7x - {}_{(+)}14 \\
\hline
 & 0
\end{array}
$$

$2x^3 + 5x^2 - 11x - 14 = (x-2)(2x^2 + 9x + 7) = (x-2)(2x^2 + 2x + 7x + 7)$

$= (x-2)[2x(x+1) + 7(x+1)] = \underline{(x-2)(x+1)(2x+7)}$

(c)

Express train	Ordinary train
D = 240 km S = x kmh^{-1} T = $[\frac{240}{x}]$ hrs	D = 240 km S = (x -12) kmh^{-1} T = $[\frac{240}{x-12}]$ hrs

Ordinary train takes one hour more

$\therefore [\frac{240}{x-12}] - \frac{240}{x} = 1$

$240\ [\frac{1}{x-1} - \frac{1}{x}]$

$\frac{x-(x-12)}{x^2-12x} = \frac{1}{240}$

$2880 = x^2 - 12x$

$0 = x^2 - 12x - 2880$

$0 = x^2 - 60x + 48x - 2880$

$0 = x(x - 60) + 48(x - 60)$

$0 = (x + 48)(x - 60)$

X – 60 = 0 OR x + 48 = 0

X = 60 kmh^{-1} OR x = -48kmh^{-1}

Ignoring the negative value:-

Speed of express train = 60 kmh^{-1}

Answer 4.

(a) $A = \begin{bmatrix} 10 & -8 \\ 5 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 1 & 3 \\ -5 & 12 \end{bmatrix}$

$X + 2A = 3B + 2I$

$X = 3B + 2I - 2A$

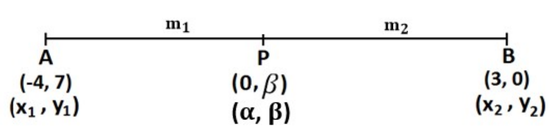
$X = 3\begin{bmatrix} 1 & 3 \\ -5 & 12 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 10 & -8 \\ 5 & 1 \end{bmatrix}$

$X = \begin{bmatrix} 3 & 9 \\ -15 & 36 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 20 & -16 \\ 10 & 2 \end{bmatrix}$

$X = \begin{bmatrix} 3+2-20 & 9+0-(-16) \\ -15+0-10 & 36+2-2 \end{bmatrix}$

$X = \begin{bmatrix} -15 & 25 \\ -25 & 36 \end{bmatrix}$

(b)



$\alpha = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$; $\beta = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

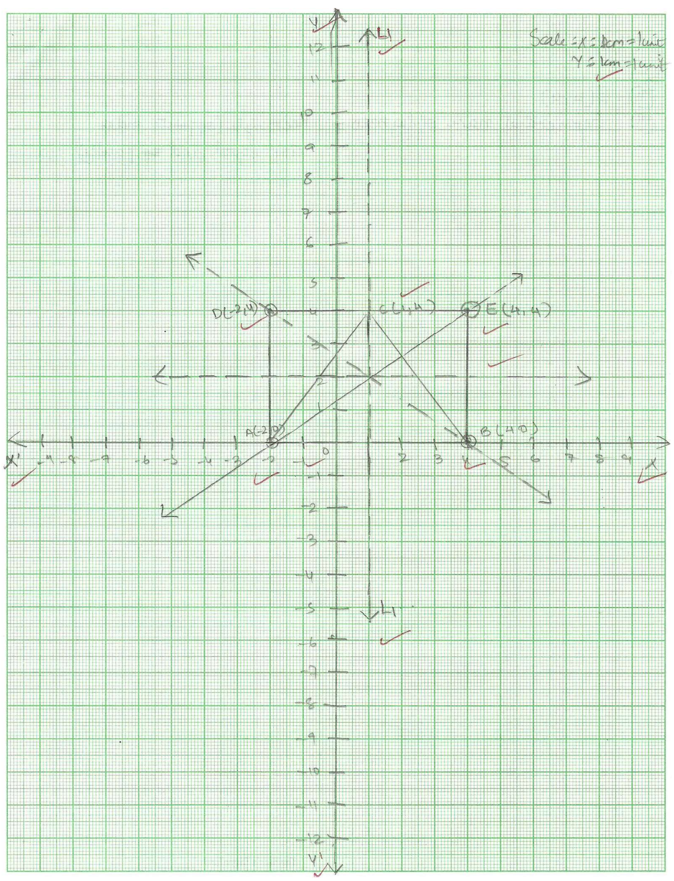
$0 = \frac{(m_1)(3) + (m_2)(-4)}{m_1 + m_2}$; $\beta = \frac{(4)(0) + (3)(7)}{4 + 3}$

$0 = 3m_1 - 4m_2$; $\beta = \frac{(4)(0) + (3)(7)}{7}$

$0 = 3m_1 - 4m_2$; $\beta = \frac{0 + 21}{7}$

$4m_2$	=	$3m_1$	;	β	=	$\frac{21}{7}$
$\frac{m_1}{m_2}$	=	$\frac{4}{3}$	;	β	=	3
$m_1:m_2$	=	$\underline{4:3}$	;	β	=	3
$P(\alpha, \beta)$	=	$\underline{(0,3)}$				

(c) graph:



i. Lines of symmetry of triangle ABC done in graph.

ii. E (4, 4)

iii. Rectangle

iv. Lines of symmetry are AE and BD.

Section II

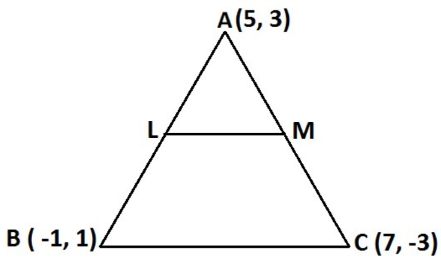
Answer 5.

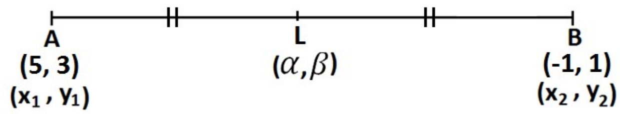
(a) Let a be the first term and d be the common difference, then

T_{10}	$= a + 9d$	$= 38$
and T_{16}	$= a + 15d$	$= 74$
Subtracting 6d	$= 36$	
d	$= \frac{36}{6}$	$= 6$
and a + 9d	$= 38$	

a + 9 × 6	= 38			
a + 54	= 38			
a	= 38 – 54			
∴ a	= – 16			
and d	= 6			
T_{31}	= a + 30d	= – 16 + 30 × 6	= – 16 + 180	= 164

(b)





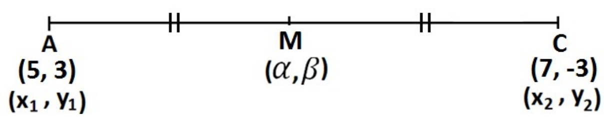
$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$\alpha = \frac{5 - 1}{2}$; $\beta = \frac{3 + 1}{2}$

$\alpha = \frac{4}{2}$; $\beta = \frac{4}{2}$

$\alpha = 2$; $\beta = 2$

L (α, β) = (2, 2)



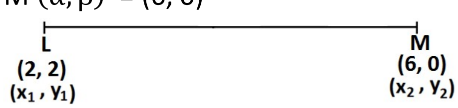
$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$\alpha = \frac{5 + 7}{2}$; $\beta = \frac{3 - 3}{2}$

$\alpha = \frac{12}{2}$; $\beta = \frac{0}{2}$

$\alpha = 6$; $\beta = 0$

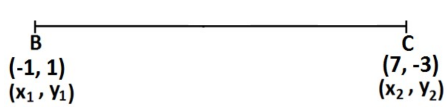
M (α, β) = (6, 0)



$LM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 2)^2 + (0 - 2)^2}$

$= \sqrt{(4)^2 + (-2)^2} = \sqrt{16 + 4}$

$= \sqrt{20} = 2\sqrt{5}$ unit



$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-(-1))^2 + (-3-1)^2} = \sqrt{(8)^2 + (-4)^2}$

$= \sqrt{64+16} = \sqrt{80} = \underline{\underline{4\sqrt{5}\text{ unit}}}$

LM $= \frac{1}{2}$ BC

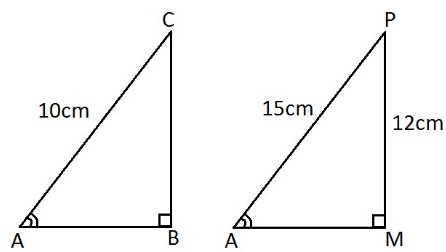
LM $= \frac{1}{2}(4\sqrt{5})$

LM $= 2\sqrt{5}$

LM $= \frac{1}{2}$ BC

<u>Hence proved.</u>

(c)



Given:
i. AC = 10 cm
ii. AP = 15 cm
iii. PM = 12 cm
To Find: i. BC
T.P.T: 1. ΔABC ~ ΔAMP

Statement	Reason
1) In ΔABC And ΔAMP, ∠CAB = ∠ PAM ∠CBA = ∠ PMA = 90° ΔABC ~ ΔAMP	 Common ∠ Given AA postulate of similarity.
2) $\frac{AC}{AP} = \frac{BC}{PM}$ $\frac{10}{15} = \frac{BC}{12}$ BC $= \frac{12\times10}{15}$ = 8 cm	corresponding sides of similar Δ are proportional.
3) $\frac{A\ \Delta ABC}{A\ \Delta AMP} = \frac{(BC)^2}{(MP)^2}$ $= \frac{8^2}{12^2} = \frac{64}{144} = \frac{4}{9} = \underline{\underline{4:9}}$	Area of similar Δ are proportional to the square of the corresponding sides.

Answer 6.

(a) Sum of three numbers in G.P. $= \frac{39}{10}$

and their product $= 1$

Let number be $\frac{a}{r}$, a, ar then

$\frac{a}{r} \times a \times ar = 1$

$a^3 = 1 = (1)^3$

$\therefore a = 1$

and $\frac{a}{r} \times a \times ar = \frac{39}{10}$

$a\left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$

$\frac{1}{r} + 1 + r \quad = \quad \frac{39}{10} \times 1 \quad = \quad \frac{39}{10}$

$r + \frac{1}{r} \quad = \quad \frac{39}{10} - 1 \quad = \quad \frac{39-10}{10} \quad = \quad \frac{29}{10}$

$r^2 + 1 \quad = \quad \frac{29}{10} r$

$10r^2 + 10 \quad = \quad 29r$

$10r^2 - 29r + 10 \quad = \quad 0$

$10r^2 - 4r - 25r + 10 \quad = \quad 0$

$2r\,(5r - 2)\,(2r - 5) \quad = \quad 0$

Either $5r - 2 \quad = \quad 0$

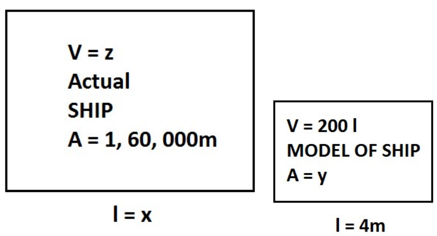
then $r \quad = \quad \frac{2}{5}$

or $2r - 5 \quad = \quad 0$

then $r \quad = \quad \frac{5}{2}$

$\therefore$ Numbers are $\frac{2}{5}, 1, \frac{4}{25}$, or $\frac{5}{2}, 1, \frac{25}{4}$

(b)



Scale factor, K $\quad = \quad 1 : 200 \quad = \quad \frac{1}{200}$

i. length of model $\quad = \quad K \times$ actual length

Actual length $\quad = \quad \frac{length\ of\ model}{k} \quad = \quad \frac{4}{\frac{1}{200}} \quad = \quad 4 \times 200 \quad = \quad \underline{\underline{800\ m}}$

ii. Area of deck on model $\quad = \quad k^2 \times$ area of deck in actual

$\therefore$ Actual area of deck $\quad = \quad \frac{Area\ of\ model}{k^2}$

$A \quad = \quad \left(\frac{1}{200}\right)^2 \times 1, 60, 000 \quad = \quad \frac{1}{40000} \times 160, 000 \quad = \quad \underline{\underline{4\ m^2}}$

iii. Vol of model $\quad = \quad K^3 \times$ vol in actual

vol of actual ship $\quad = \quad \frac{vol.of\ model}{k^3}$

$= \quad \frac{200}{\frac{1}{(200)^3}} \quad = \quad 200 \times (200)^3$

$= \quad (200)^4 \quad = \quad \underline{\underline{16, 00, 000\ m^3}}$

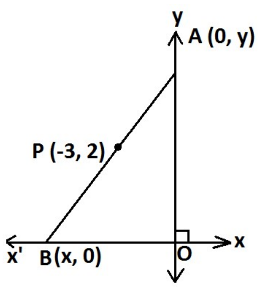
(c) Fv = Rs. 100

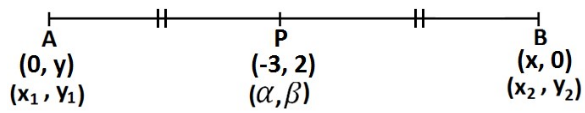
n = ?

I	=	mv × n		
mv	=	Rs. 150		
I	=	Rs. 9000		
9000	=	150 × n		
∴ n	=	60		
D	=	?		
D	=	$\frac{Fv \times n \times r}{100}$		
	=	$\frac{100 \times 60 \times 6}{100}$		
SP	=	Rs. 200	=	360
Sale proceed	=	30 × 200	=	Rs. 6,000

Answer 7.

(a)





$$\alpha = \frac{x_1 + x_2}{2} \quad ; \quad \beta = \frac{y_1 + y_2}{2}$$

$$-3 = \frac{0 + x}{2} \quad ; \quad 2 = \frac{y + 0}{2}$$

$$-6 = 0 + x \quad ; \quad 4 = y + 0$$

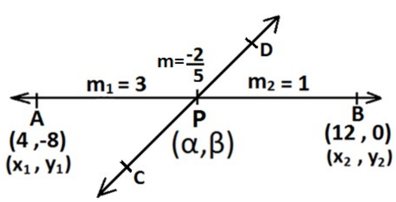
$$x = -6 - 0; \quad y = 4 - 0$$

$$x = -6 \quad ; \quad y = 4$$

A = (0, 4)

B = (– 6, 0)

(b)



$$\alpha = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \quad ; \quad \beta = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\alpha = \frac{(3 \times 12) + (1 \times 4)}{3 + 1} \quad ; \quad \beta = \frac{(3 \times 0) + (1 \times -8)}{3 + 1}$$

$$\alpha = \frac{36 + 4}{4} \quad ; \quad \beta = \frac{0 - 8}{4}$$

$$\alpha = \frac{40}{4} \quad ; \quad \beta = \frac{-8}{4}$$

$$\alpha = 10 \quad ; \quad \beta = -2$$

P (α, β) = (10, - 2)

Equation of line CD:

$$P = (10, -2) = (x_1, y_1)$$

$$m = \frac{-2}{5}$$

$$m = \frac{y - y_1}{x - x_1}$$

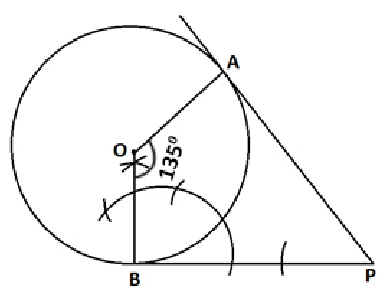
$$\frac{-2}{5} = \frac{(y) - (-2)}{(x) - (10)}$$

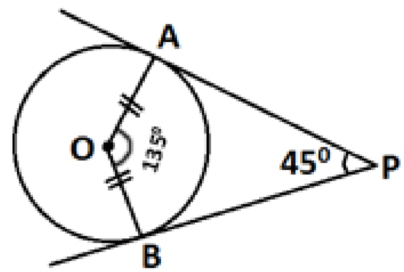
$$\frac{-2}{5} = \frac{y + 2}{x - 10}$$

$$20 - 10 = 5y + 2x$$

$\underline{\underline{2x + 5y = 10}}$

(c)

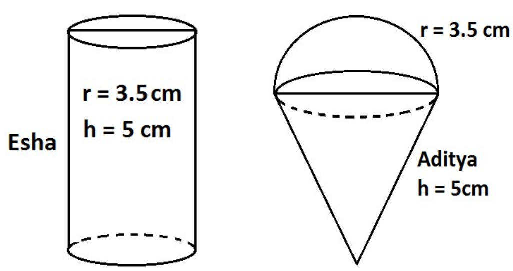




PA = PB = $\underline{\underline{7.5\text{cm}}}$

Answer 8.

(a)



i. Volume Esha got (cylinder) = $\pi r^2 h = \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times 5 = \underline{\underline{192.5\text{cm}^3}}$

Volume Aditya got = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 + (2r + h)$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5^2 (7 + 5) = 12.83 (12)$

$= \underline{\underline{154\text{cm}^3}}$

ii. Cost of Aditya = 154cm^3 = Rs. 40

Esha: 192.5cm^3 = ?

$= \frac{192.50 \times 40}{154} = \underline{\underline{\text{Rs. } 50}}$

(b)

Weight	No. of chocolate	Fx (b)
50-60	30	30
60-70	50	80
70-80	70	150
80-90	100	250
90-100	80	330
100-110	60	390
110-120	10	400
Total	400	

n = 400 even

i. Me = x value of $\left(\frac{n}{2}\right)^{th}$ term = x value of $\left(\frac{400}{2}\right)^{th}$ term

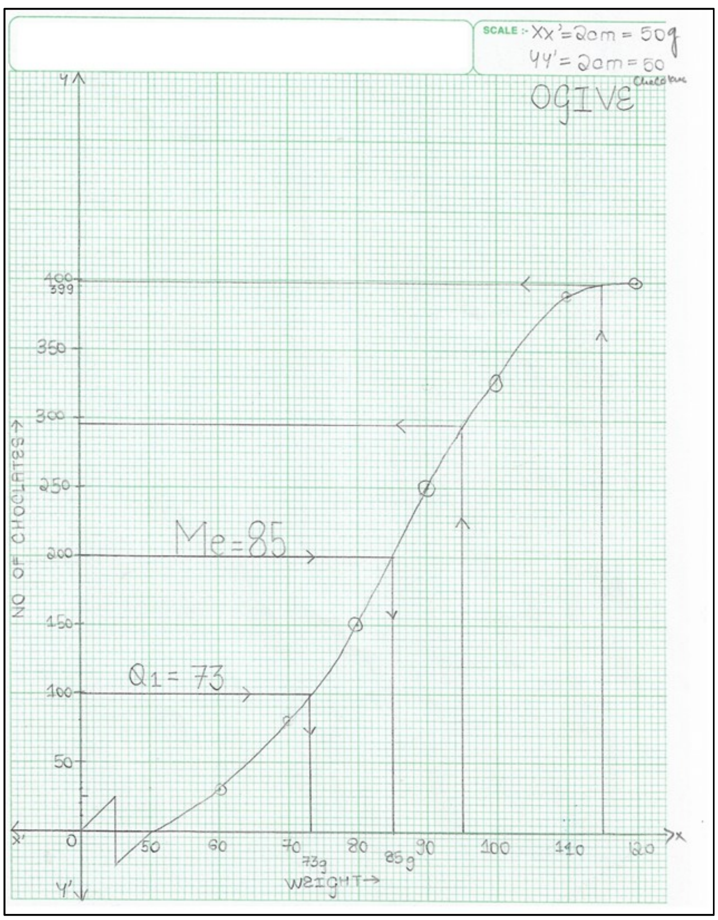
= x value of 200^{th} term = <u>85 g</u>

ii. Q_1 = x value of $\left(\frac{n}{4}\right)^{th}$ term = x value of $\left(\frac{400}{4}\right)^{th}$ term

= x value of 100^{th} term = <u>73 g</u>

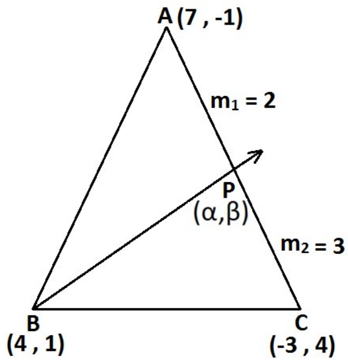
iii. No. of chocolates = <u>295 chocolates</u>

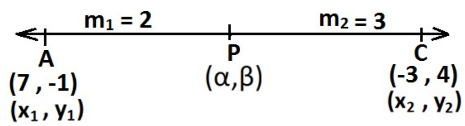
iv. No. of chocolates = 400 – 399 = <u>1 chocolate</u>



Answer 9.

(a)





$\alpha = \dfrac{m_1x_2 + m_2x_1}{m_1+m_2}$; $\beta = \dfrac{m_1y_2 + m_2y_1}{m_1+m_2}$

$\alpha = \dfrac{(2 \times -3)+(3 \times 7)}{2 + 3}$; $\beta = \dfrac{(2 \times 4)+(3 \times 7)}{2 + 3}$

$\alpha = \dfrac{-6 + 21}{5}$; $\beta = \dfrac{8 - 3}{5}$

$\alpha = \dfrac{15}{5}$; $\beta = \dfrac{5}{5}$

$\alpha = \underline{\underline{3}}$; $\beta = \underline{\underline{1}}$

P (α,β) = $\underline{\underline{(3, 1)}}$

∴ Equation of line BP:

B = (4, 1) = (x_1, y_1)

P = (3, 1) = (x_2, y_2)

$\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y - y_1}{x - x_1}$

$\dfrac{(1) - (1)}{(3) - (4)} = \dfrac{(y) - (1)}{(x) - (4)}$

$\dfrac{0}{-1} = \dfrac{y - 1}{x - 4}$

∴ 0 = -1 (y - 1)

0 = - y + 1

∴ $\underline{\underline{y}}$ = $\underline{\underline{1}}$

(b)

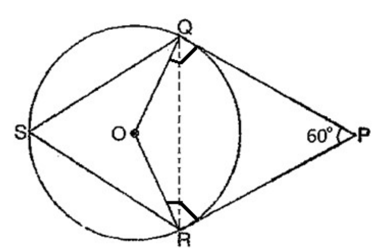
Given:

i. QPR = 60° ii. PQ and PR are tangents.

To find:

i. ∠QOR ii. ∠OQR iii. ∠QSR

Construction: Join QR.



Statement		Reason
1.	∠RPQ = 60°	Given.
2.	∠OQP = ∠ORP = 90°	Tangent is perpendicular to the radius at the point of contact.
3.	∠ROQ = 360°– [90°+ 90°+ 60°]	Sum of the angles of a quadrilateral

	$= 360° - 240° = \underline{\underline{120°}}$	OQPR is 360°.
4.	$\angle RSQ = \frac{1}{2}\angle ROQ$ $= \frac{1}{2} \times 120° = \underline{\underline{60°}}$	Angle subtended at the centre is double that of the circumference.
5.	OQ = OR	Radii of the same circle.
6.	$\angle OQR = \angle ORQ$ $= \frac{180° - 120°}{2} = \frac{60°}{2} = \underline{\underline{30°}}$	Sum of the angles of a triangle is 180° & angles opp. equal sides are equal.

(c) L.H.S.

$$\frac{\text{cosec A} - 1}{\text{cosec A} + 1} = \frac{\frac{1}{sin\,A} - 1}{\frac{1}{sin\,A} + 1} = \frac{1 - sin\,A}{sin\,A} \times \frac{sin\,A}{1 + sin\,A}$$

$$= \frac{1 - sin\,A}{1 + sin\,A} \times \frac{1 + sin\,A}{1 + sin\,A} = \frac{1 - sin^2\,A}{(1 + sin\,A)^2}$$

$$= \frac{cos^2\,A}{(1 + sin\,A)^2} = \left(\frac{cos\,A}{1 + sin\,A}\right)^2$$

= R.H.S

$$\left(\frac{cos\,A}{1 + sin\,A}\right)^2$$

L.H.S = R.H.S

Hence proved.

Answer 10.

(a)

Marks	Cf	f	x	A = 25 D = x – A	t	ft
0 – 10	7	7	5	-20	-2	-14
10 – 20	19	12	12	-10	-1	-12
20 – 30	32	13	25	0	0	0
30 – 40	42	10	35	10	1	10
40 – 50	50	8	45	20	2	16
Total	$\sum f$ = 150					$\sum f\,t$ = -26 + 26 = 0

$\bar{x} = A + \left(\frac{\sum ft}{\sum t} \times i\right)$

$\bar{x} = 25 + \left(\frac{0}{150} \times 10\right)$

$\bar{x} = 25 + 0$

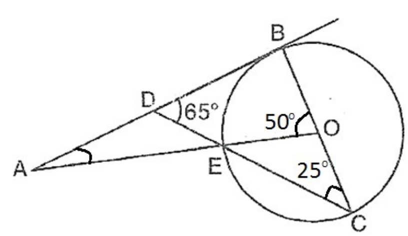
$\bar{x} = \underline{\underline{25}}$

(b) Given:

i. ∠BDC = 65°

ii. AB is a tangent.

To Find: ∠BAO



	Statement		Reason
1.	∠BDC	= 65°	Given.
2.	∠OBD	= 90°	Radius is perpendicular to the tangent at the point of the contact.
3.	∠BCD	= 180° – (90° + 65°) = 180° – 155° = 25°	Sum of the angles of a triangle BCD is 180°.
4.	∠BOE	= 2 ∠BCE = 2 × 25° = 50°	Angle subtended at the centre is double that of the circumference.
5. i.	In Δ BOA, ∠BAO	= 180° – (90° + 50°) = 180° – 140° = 40°	Sum of the angles of a triangle is 180°.

(c) LHS: = $\frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{\sin A\,(1+\cos A)}$ = $\frac{1 - cos^2 A + 1 + \cos^2 A + 2\cos A}{(1+\cos A)\sin A}$

= $\frac{2 + 2\cos A}{\sin A\,(1+\cos A)}$ = $\frac{2\,(1+\cos A)}{(1+cosA)(sin\ A)}$

= $\frac{2}{\sin A}$ = 2 cosec A

LHS = RHS

Hence proved

Answer 11.

(a) Total outcomes = 6 = {1, 2, 3, 4, 5, 6}

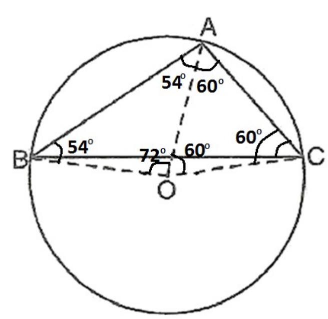
i. Favourable outcomes = 0 ;

P(7 = 0) = $\frac{0}{6}$ = 0

ii. Favourable 0utcomes = 6 = {1, 2, 3, 4, 5, 6}

$\frac{Favourable\ no.of\ outcomes}{Total\ no.of\ outcomes}$ = $\frac{6}{6}$ = 1

(b) Given:



i. AB is the side of a regular pentagon.

ii. AC is the side of a regular hexagon.

To find:

i. ∠BAC

ii. ∠ABC

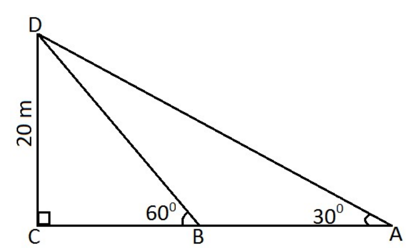
iii. ∠ACB

Cons : Join AO, BO, CO.

	Statement	Reason
1.	AB is the side of regular pentagon.	Given.

2.	Each interior angle of a regular pentagon $= \frac{(2n-4)\times 90°}{n}$ $= \frac{(2\times 5-4)\ 90°}{5} = 108°$	By formula.
3.	∠OBA $=$ ∠OAB $= \frac{108°}{2} = 54°$	OA and OB are angle bisectors of each interior angle.
4.	∠AOB $= 180° - (54° + 54°)$ $= 180° - 108° = 72°$	Sum of the angles of a triangle AOB is 180°.
5.	∠ACB $= \frac{1}{2}$ ∠AOB $= \frac{1}{2} \times 72° = \underline{\underline{36°}}$	Angle subtended at the centre is double that of the circumference.
6.	AC is the side of regular hexagon.	Given.
7.	Each interior angle of a regular hexagon $= \frac{(2n-4)\times 90°}{n}$ $= \frac{(2\times 6-4)\ 90°}{6} = \frac{720}{6} = 120°$	By formula.
8.	∴ ∠OAC $=$ ∠OCA $= \frac{120°}{2} = 60°$	OA and OC are angle bisectors of each interior angle.
9.	∠ AOC $= 180° - (60° + 60°)$ $= 180° - 120° = 60°$	Sum of the angles of a triangle AOC is 180°.
10.	∠ABC $= \frac{1}{2} \times$ ∠AOC $= \frac{1}{2} \times 60° = \underline{\underline{30°}}$	Angle subtended at the centre is double that of the circumference.
11.	∠BAC $=$ ∠BAO + ∠OAC $= 54° + 60° = \underline{\underline{114°}}$	By addition property.

(c) In ΔDCB,



$\tan 60° = \frac{DC}{CB}$

$\sqrt{3} = \frac{20}{CD} = \frac{20}{\sqrt{3}}$

$= \frac{20\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{20\sqrt{3}}{3}$

$CB = 20\sqrt{3} \div 3 = 20 \times 1.732 \div 3$

$= 34.64\text{ m} \div 3 = 11.54\text{ m}$

In ΔDCA

$\tan 30° = \frac{DC}{AC}$

$\frac{1}{\sqrt{3}} = \frac{20}{AC}$

$AC = 20\sqrt{3} = 20 \times 1.732 = \underline{34.64\text{ m}}$

$AB = AC - CB = 34.64 - 11.54 = 23.10 = \underline{\underline{23\text{m}}}$

Answers of Practice Paper 9

Section I

Answer 1.

(a) F.V = Rs.100

n = 62 shares

r = 7.5%

M.V. = Rs.132

I = ?

D = ?

R = ?

i. Investment = M.V. × Number of shares = 132 × 62 = <u>Rs. 8184</u>

ii. Annual income = $\frac{F.V. \times n \times r}{100}$ = $\frac{100 \times 62 \times 75}{100 \times 10}$ = <u>Rs. 465</u>

iii. Extra income = 150

D = $\frac{F.V. \times n \times r}{100}$

150 = $\frac{100 \times n \times 7.5}{100}$

n = <u>20 shares</u>

(b)

{No. of possible. events /Sample space (n)} = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

i. No of favorable events (m) = {(1, 1), (2, 2), (3, 3) (4, 4), (5, 5), (6, 6)}
= 6

∴ {Probability of getting the same number of 9 on both the die [P(E)]} = $\frac{m}{n}$ = $\frac{6}{36}$ = $\frac{1}{6}$

ii. No of favorable events (m) = {(3, 6) (6, 3), (4, 5), (5, 4)} = 4

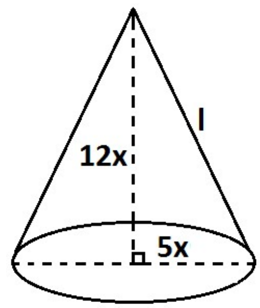
∴ {Probability getting a total sum of 9 on the die p (E)]} = $\frac{4}{36}$ = $\frac{1}{9}$

iii. No of favourable events [m] = {(2, 4), (4, 2)} = (2)

∴ {Probability of getting product of 8, [p(E)]} $= \frac{m}{n} = \frac{2}{36} = \frac{1}{18}$

(c) Volume $= 2512cm^3$

$\pi = 3.14$



Volume $= \frac{1}{3}\pi r^2 h$

$2512 = \frac{1}{3} \times \frac{314}{100} \times 25x^2 \times 12x$

$2512 = \frac{1}{3} \times \frac{314}{100} \times 300x^3$

$300x^3 = \frac{2512 \times 3 \times 100}{314}$

$x^3 = 8$

$x = \sqrt[3]{8}$

x = 2cm

radius = 5x = 5 × 2 = <u>10 cm</u>

Height = 12x = 12 × 2 = 24 cm

By Pythagoras theorem,

$l^2 = h^2 + r^2$

$l = \sqrt{h^2 + r^2}$

$l = \sqrt{(24)^2 + (10)^2}$

$l = \sqrt{576 + 100}$

$l = \sqrt{676}$

l = <u>26 cm</u>

Answer 2.

(a) ∴ L.H.S

$= \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \sqrt{\frac{1 - \cos}{1 + \cos A}} \times \sqrt{\frac{1 - \cos A}{1 - \cos A}}$

$= \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} = \sqrt{\frac{(1 - co \ \ A)^2}{\sin A}}$

$= \frac{1 - \cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

= cosec A – cot A

R.H.S

= cosec A – cot A

<u>L.H.S = R.H.S</u>

<u>Hence proved</u>

(b) $\frac{\sqrt{a^2+b^2} + \sqrt{a^2-b^2}}{\sqrt{a^2+b^2} - \sqrt{a^2-b^2}} = x$

Using Componendo of dividend

$$\frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}+\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}-\sqrt{a^2+b^2}+\sqrt{a^2-b^2}} = \frac{x+1}{x-1}$$

$$\frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2-b^2}} = \frac{x+1}{x-1}$$

On squaring both the sides, we get:-

$$\frac{a^2+b^2}{a^2-b^2} = \frac{(x+1)^2}{(x-1)^2}$$

$\therefore$ $a^2 (x^2 - 2x + 1) + b^2 (x^2 - 2x + 1) = a^2 (x^2 + 2x + 1) - b^2(x^2 + 2x + 1)$

$\therefore$ $a^2x^2 - 2a^2x + a^2 + b^2x^2 - 2b^2x + b^2 = a^2 x^2 + 2a^2x + a^2 - b^2x^2 - 2b^2x + b^2$

$2 x^2 b^2 + 2 b^2 = 4a^2x$

$2 (x^2 b^2 + b^2) = 24 a^2x$

$x^2 b^2 + b^2 = 2a^2x$

$b^2 (x^2 + 1) = 2a^2x$

$b^2 (x^2+1) = \frac{2a^2x}{x^2+1}$

Hence proved.

(c) (x – 2) is factor $\therefore$ x – 2 = 0, x = 2

f(x) = $x^3 - 2x^2 - px + 18$

f(2) = 8 – 8 – 2 p + 18

since (x – 2) is a factor, the remainder is 0

0 = –2p + 18

2p = 18,

p = 19

$x^2 + 0 - 9$

$\therefore$ $x-2\,\overline{)\,x^3 - 2x^2 - 9x + 18}$

$_{(-)}x^3 +_{(-)} 2x^2$

0 – 9x

$_{(-)}$ 0 – $_{(+)}$ 0

-9x -18

$_{(+)}$ -9x $_{(-)}$+ 18

0 0

$x^3 - 2x^2 - px + 18 \quad = \quad (x-2)(x^2-9) \quad = \quad \underline{(x-2)(x+3)(x-3)}$

Answer 3.

(a) v = 2500, n = 24 months, mv = 66, 250

i. mv = nx + I

66250 = (2500 X 24) + I

I = RS. 6250

ii. $I = \frac{n\,X\,r\,(n+1)}{2400}$

$6250 = \frac{24\,X\,25\,X\,2500\,X\,r}{2400}$

r = 10%

(b) Let A_1 and A_2, A_3 be three arithmetic mean between 15 and 27.

∴ 15, A_1, A_2, A_3, 27 are in A.P.

∴ $T_1 = a = 15$

$T_5 = 27$

$a + 4d = 27$

$15 + 4d = 27$

$4d = 27 - 15 = 12$

$d = \frac{12}{4}$

$d = 3$

∴ $A_1 = 15 + 3 = \underline{18}$

$A_2 = 18 + 3 = \underline{21}$

$A_3 = 21 + 3 = \underline{24}$

(c)

Class	F	m	Fm
0 – 20	5	10	50
20 – 40	f_1	30	$30f_1$
40 – 60	10	50	500
60 – 80	f_2	70	$70f_2$
80 – 100	7	90	630
100 – 120	8	110	880
Total	$(30 + f_1 + f_2)$		$2060 + 30f_1 + 70f_2$

Total f = 50 (given)

Mean = 62.8

$\text{Mean} = \frac{\Sigma fm}{\Sigma f}$

$62.8 = \frac{2060+30f_1+70f_2}{50}$

$108 = 3f_1 + 7f_2$ ..... (i)

$30 + f_1 + f_2 = 50$

$f_1 + f_2 = 20$ ... (ii)

Solving eq (i) & (ii)

$3f_1 + 7f_2 = 108$

$3f_1 + 3f_2 = 60$

$-\quad -\qquad -$

$4f_2 = 48$

$\underline{\underline{f_2 = 12}}$

$f_1 + f_2 = 20$

$\underline{\underline{f_1 = 8}}$

Answer 4.

(a) Scale $= 1 : 200$

Model length $= \frac{1}{k}$ X actual length

$4 = \frac{1}{k}$ X actual length

$4 = \frac{1}{200}$ X actual l

$\underline{\underline{800\text{m}}} = \underline{\underline{\text{actual } l}}$

Model Vol. $= \frac{1}{k^3}$ X actual vol. $= \frac{1}{(200)^3}$ X actual vol.

200 X 1000 X 200 X 200 X 200 = actual vol.

16×10^{11} cm^3 = actual vol.

$\frac{16 \times 10^{11}}{10^6}$ m^3 = actual vol.

$\underline{\underline{16 \times 10^5 \text{ m}^3}}$ = $\underline{\underline{\text{actual vol.}}}$

(b) $S_n = 5641$ and G.P. is 1 + 4 + 16 + 64 + ...

Here a = 1,

r = 4 (r > 1)

$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$

$5641 = \frac{1(4^n - 1)}{4 - 1}$

$= \frac{4^n - 1}{3}$

4	16384
4	4096
4	1024
4	256
4	64
4	16
4	4
	1

$4^n - 1 \quad = \quad 5641 \times 3 \quad = \quad 16383$

$4^n \quad = \quad 16383 + 1 \quad = \quad 16384$

$4^n \quad = \quad 4^7$

Comparing we get,

n = $\underline{\underline{7}}$

∴ There are 7 terms in the given G.P.

(c) If a $= \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

a $= \frac{2 \times 2 \times \sqrt{3} \times \sqrt{2}}{\sqrt{2}+\sqrt{3}}$

$\frac{a}{2\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{3}}$ …….. (i)

$\frac{a+2\sqrt{3}}{a-2\sqrt{3}} = \frac{\frac{a}{2\sqrt{3}}+1}{\frac{a}{2\sqrt{3}}-1}$ (divide numerator and denominator by $2\sqrt{3}$)

$= \frac{\frac{2\sqrt{2}}{\sqrt{2}+\sqrt{3}}+1}{\frac{2\sqrt{2}}{\sqrt{2}+\sqrt{3}}-1}$

$= \frac{2\sqrt{2}+(\sqrt{2}+\sqrt{3})}{2\sqrt{2}-(\sqrt{2}+\sqrt{3})}$

$= \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$ …….. (ii)

a $= \frac{2 \times 2 \times \sqrt{3} \times \sqrt{2}}{\sqrt{2}+\sqrt{3}}$

$\frac{a}{2\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}}$ …….. (iii)

$\frac{a+2\sqrt{2}}{a-2\sqrt{2}} = \frac{\frac{a}{2\sqrt{2}}+1}{\frac{a}{2\sqrt{2}}-1}$ (divide numerator and denominator by $2\sqrt{2}$)

$= \frac{\frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}}+1}{\frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}}-1}$

$= \frac{2\sqrt{3}+(\sqrt{2}+\sqrt{3})}{2\sqrt{3}-(\sqrt{2}+\sqrt{3})}$

$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ …….. (iv)

Consider $\frac{a+2\sqrt{2}}{a-2\sqrt{2}} + \frac{a+2\sqrt{3}}{a-2\sqrt{3}}$

$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$ from (ii) and (iv)

$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

$= \frac{3\sqrt{3}+\sqrt{2}-3\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

$= \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{2(\sqrt{3}-2\sqrt{2})}{(\sqrt{3}-2\sqrt{2})}$

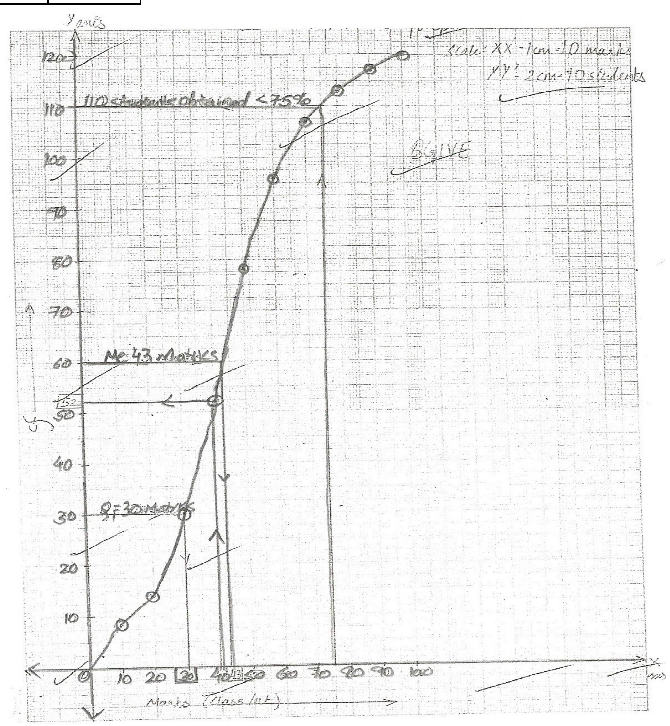
$= \underline{\underline{2}}$

Section II

Answer 5.

(a)

Marks	F	Cf
0-10	9	9
10-20	5	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120
Total	120	



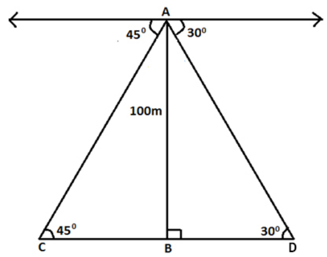
i. Median = x value of $\left[\frac{n}{2}\right]^{th}$ term = x value of 60^{th} term = 43 marks

ii. Q_1 = x value of $\left[\frac{n}{4}\right]^{th}$ term = x value of 30^{th} term = 30 marks

iii. Students who obtained more than 75% = 120 – 110 = 10 students

iv. No. of students who failed if 40% was passing = 52 students.

(b)



In Δ ABC, tan 45° $= \frac{AB}{BC}$

BC = 100m

In Δ ABD, tan 30° $= \frac{AB}{BD}$, BD $= \sqrt{3}$m

∴ Distance between 2 ships = BC + BD

$= 100\text{m} + 100\sqrt{3}\text{m} = 100(1 + \sqrt{3})$

$= 100(1 + 1.732) = 100(2.732)$

= <u>273.20m</u>

Answer 6.

(a) $-\frac{4}{3} \leq 2\langle\frac{x}{4} + 1\rangle - \frac{4}{3} < \frac{5}{6}$, x∈R

$-\frac{4}{3} \leq 2\langle\frac{x+4}{4}\rangle - \frac{4}{3}$; $2\,\frac{(x+4)}{4} - \frac{4}{3} < \frac{5}{6}$

$-\frac{4}{3} \leq \frac{3\,(x+4)-8}{6}$; $\frac{3\,(x+4)-8}{6} < \frac{5}{6}$

$-8 \leq 3x + 12 - 8$; $3x + 12 - 8 < 5$

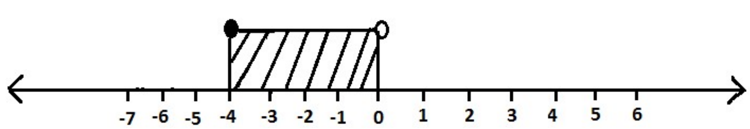
$-8 \leq 3x + 4$; $3x + 4 < 5$

$-12 \leq 3x$; $3x < 1$

$-4 \leq x$; $x < \frac{1}{3}$

$-4 \leq x < \frac{1}{3}$

S.S. (x) = $\left\{x : -4 \leq x < \frac{1}{3}, x \in R\right\}$



(b) AB = BC

$$\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x & -y \\ z & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-0 & -2-3 \\ 1+0 & -1+1 \end{bmatrix} = \begin{bmatrix} x-z & -y+0 \\ z & 0 \end{bmatrix}$$

$$\begin{bmatrix}2 & -5\\1 & 0\end{bmatrix} = \begin{bmatrix}x-z & -y+0\\z & 0\end{bmatrix}$$

∴ $\underline{\underline{z}}$ = $\underline{\underline{1}}$

2 = x – z

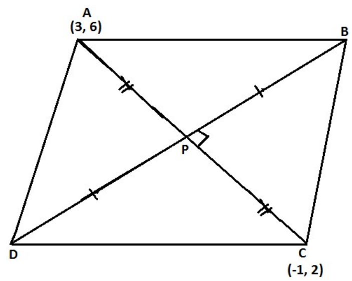
2 + 1 = x

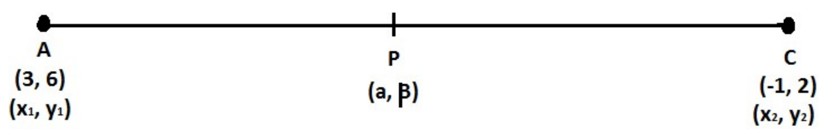
$\underline{\underline{x}}$ = $\underline{\underline{3}}$

-y = -5

$\underline{\underline{y}}$ = $\underline{\underline{5}}$

(c)





$\alpha = \frac{x_1+x_2}{2}$; $\beta = \frac{y_1+y_2}{2}$

$\alpha = \frac{3-1}{2}$; $\beta = \frac{6+2}{2}$

$\underline{\alpha = 1}$; $\underline{\beta = 4}$

$P(\alpha, \beta)$ = (1, 4)

m of line AC = $\frac{y_2-y_1}{x_2-x_1}$ = $\frac{2-6}{-1-3}$ = $\frac{-4}{-4}$

m of line AC = 1

m of line BD = $\frac{-1}{\text{m of AC}}$ = -1

∴ Equation of line BD; m = $\frac{y-y_1}{x-x_1}$

-1 = $\frac{y-4}{x-1}$

-x + 1 = y – 4

5 = x + y

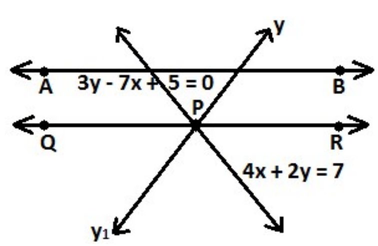
$\underline{\underline{Y}}$ = $\underline{\underline{5-x}}$

Answer 7.

(a) P (x, y) = (0, y), equation: 7 = 4x + 2y

Substituting P (0, y) in the equation:

4(0) + 2y = 7



$2y = 7$

$y = 3.5 = \frac{7}{2} = 3\frac{1}{2}$

m of line AB = m' of line PQ

m of AB:

$3y - 7x = -5$

$3y + 5 = 7x$

$3y = 7x - 5$

$y = \frac{7x}{3} - \frac{5}{3}$

$m = \frac{7}{3}$

$\therefore$ m of line PQ = $\frac{7}{3}$

Equation of line PQ:

P (0, 7) = x_1, y_1

$m = \frac{7}{3}$

$m = \frac{y - y_1}{x - x_1}$

$\frac{7}{3} = \left[y - \frac{7}{2}\right] \div [x - 0]$

$\frac{7}{3} = \left[\frac{2y-7}{2x}\right]$

$14x = 6y - 21$

$14x + 21 = 6y$

$\frac{14x}{3} + \frac{21}{6} = y$

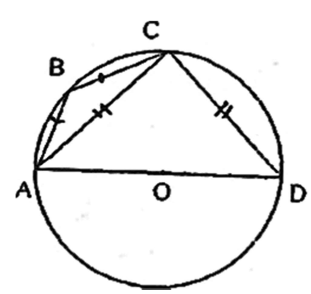
$\frac{7x}{3} + \frac{7}{2} = y$

(b) <u>Given:</u>

i. O is the center.

ii. AB = BC

iii. AC = CD



<u>To Find:</u>

i. ∠ ABC

ii. ∠ BAD

STATEMENT	REASON
1] ∠ACD = 90°	∠ in a semi-circle
2] ∠CAD = ∠ADC = x	Given

3] ∴ In Δ ACD:- ∠CAD + ∠ACD + ∠ADC = 180° 2x + 90° =180° x = 45°	Sum of all angles of a triangle is 180°
4] ∠ ABC = 180° - ∠CDA a = 180° - 45° a = 135°	Opposite angles of a cyclic quadrilateral are supplementary.
5] ∠BAC = ∠BCA = y°	Given
6] In ΔABC, ∠ BAC + ∠BCA + ∠ABC = 180° 2x + 145° = 180° x = 22.50°	Sum of all angles in a triangle = 180°
7] ∴ ∠ BAD = ∠ BAC + ∠CAD b = 22.5° + 45° b = 67.50°	Addition property.

(c)

$2x^2 - 12x + 5 = 0$

a = 2, b = -12, c = 5

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

x = $\frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(5)}}{2(2)}$

X = $\frac{12 \pm \sqrt{144 - 40}}{4}$

x = $\frac{12 \pm \sqrt{104}}{4}$

x = $\frac{2(6 \pm \sqrt{26})}{4}$

x = $\frac{6 + \sqrt{26}}{2}$ or x = $\frac{6 - \sqrt{26}}{2}$

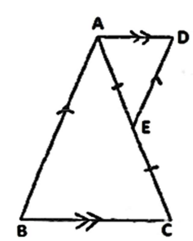
x = $\frac{6 + 5.099}{2}$ or x = $\frac{6 - 5.099}{2}$

x = 5.5495 or x = 0.4505

x = 5.55 or x = 0.45

S.S (x) = {5.55, 0.45}

Answer 8.

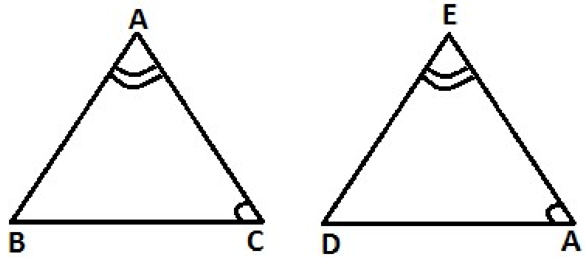


(a) Given:

i. E is the mid-point of AC

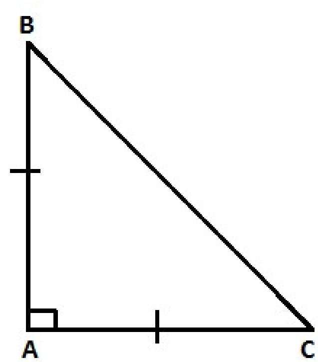
ii. Area of triangle ABC is 200cm^2

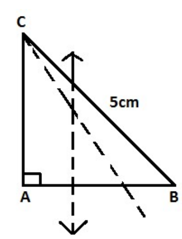
<u>To find:</u> Area of triangle ADE.



STATEMENT	REASON
1] In ΔABC, ΔADE, i) ∠BAC = ∠AED ii) ∠ACB = ∠DAE iii) ∴ ΔABC ~ ΔAED $\frac{AB}{DE} = \frac{BC}{AD} = \frac{AC}{AE}$ $\frac{AC}{AE} = \frac{2x}{x} = \frac{2}{1}$	-Alternate angles are equal -Alternate angles are equal -By 'AA' test of similarity corresponding sides of similar triangles.
2] $\frac{A\Delta ABC}{A\Delta AED} = \frac{(AC)^2}{(AE)^2} = \frac{(2)^2}{(1)^2}$ $\frac{200cm^2}{A\Delta AED} = \frac{4}{1}$ $\Delta AED = \frac{200}{4}$ = 50 cm^2	When two triangles are similar the ratio of their areas is equal to the square of the lengths of their corresponding segment.

(b)





BC = 5 CM = hypotenuse

AB = AC = X

$(AB)^2 + (AC)^2 = (BC)^2$

$x^2 + x^2 = 25$

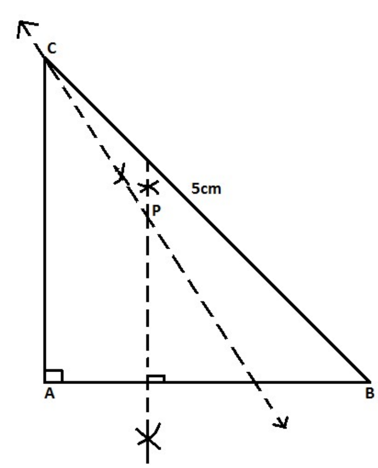
$2x^2 = 25$

$x = \sqrt{\frac{25}{2}} = \frac{5\sqrt{2}}{2}$

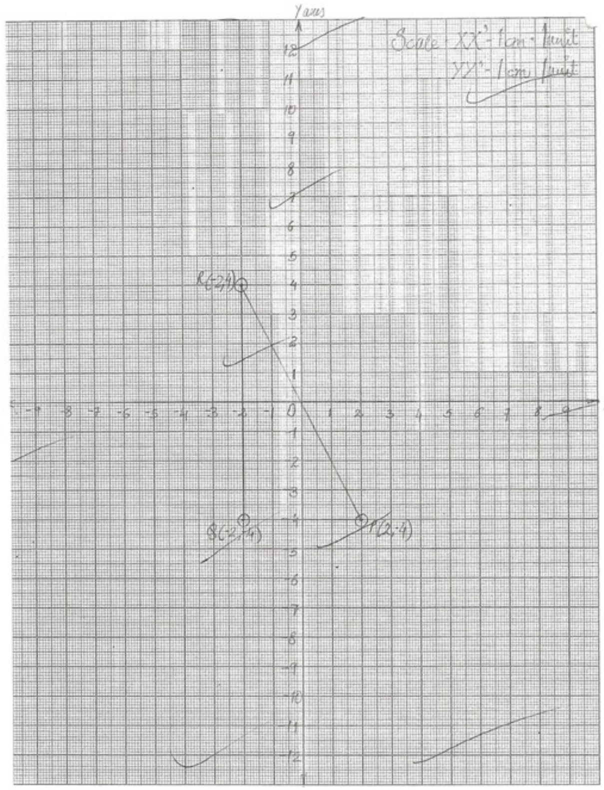
x = 2.5 X 1.414

<u>x</u> = <u>3.5350 cm</u>

Construction:-



(c)



i. P (2, -4) $\xrightarrow{M_y}$ Q (- 2, -4)

ii. Q (-2, -4) $\xrightarrow{M_x}$ R (- 2, 4)

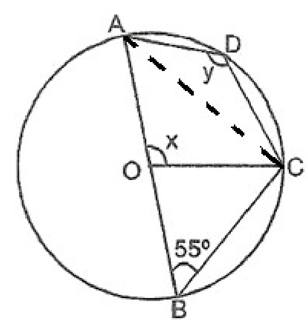
iii. Right –angled triangle

iv. A $= \frac{1}{2}$ X b X h

$= \frac{1}{2}$ X 4 X 8

= <u>16 sq. Units</u>

Answer 9.



(a) Given: ∠ABC = 55°

To find:

i. x

ii. y

Const. : Join AC.

Statement		Reason
1.	∠ABC = 55°	Given.
2.	∠y = 180° – ∠ABC = 180° – 55° = 125°	Opposite angles of a cyclic quadrilateral ABCD are supplementary.
3.	∠AOC = 2 ∠ABC x = 2 × 55° x = 110°	The angle which an arc subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

(b) Let the speed of stream be x kmph

Downstream	Upstream
d = 12 km	d = 12 km
s = 9 + x km / h	s = 9 – x km / h

t = $\frac{d}{s}$ = $\frac{12}{9+x}$ h	t = $\frac{d}{s}$ = $\frac{12}{9-x}$ h

The equation is

$$\frac{12}{9+x} + \frac{12}{9-x} = 3$$

$$\frac{108-12x+108+12x}{81-x^2} = 3$$

$$216 = 243 - 3x^2$$

$$3x^2 = 27$$

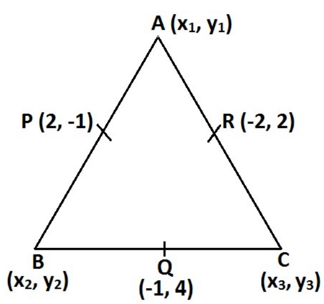
$$x^2 = 9$$

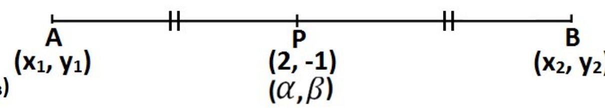
$$x = \sqrt{9} = \pm 3$$

Ignoring-v sign, x = 3 km/h

∴ speed of the stream = 3 km/h

(c)

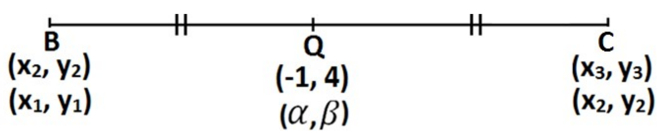




$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$2 \times 2 = x_1 + x_2$; $-1 \times 2 = y_1 + y_2$

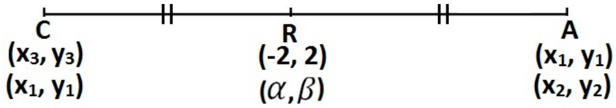
$4 = x_1 + x_2$ ... (i) ; $-2 = y_1 + y_2$ ... (ii)



$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$-1 \times 2 = x_2 + x_3$; $4 \times 2 = y_2 + y_3$

$-2 = x_2 + x_3$ ... (iii) ; $8 = y_2 + y_3$ ... (iv)



$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$-2 \times 2 = x_3 + x_1$; $2 \times 2 = y_3 + y_1$

$-4 = x_3 + x_1$ ... (v) ; $4 = y_3 + y_1$ ... (vi)

Solving equation (i), (iii), (v)

$$\begin{aligned} x_1 + x_2 &= 4 \text{ ----- (i)} \\ x_2 + x_3 &= -2 \text{ ----- (iii)} \\ x_1 + x_3 &= -4 \text{ ------(v)} \\ \hline 2x_1 + 2x_2 + 2x_3 &= -2 \end{aligned}$$

$2(x_1 + x_2 + x_3) = -2$

$x_1 + x_2 + x_3 = -1$

$x_1 = (x_1 + x_2 + x_3) - (x_2 + x_3) = (-1) - (-2) = -1 + 2 = \underline{\underline{1}}$

$x_2 = (x_1 + x_2 + x_3) - (x_1 + x_3) = (-1) - (-4) = -1 + 4 = \underline{\underline{3}}$

$x_3 = (x_1 + x_2 + x_3) - (x_1 + x_2) = (-1) - (4) = -1 - 4 = \underline{\underline{-5}}$

Solving equation (ii), (iv) and (vi)

$$\begin{aligned} y_1 + y_2 &= -2 \\ y_2 + y_3 &= 8 \\ y_3 + y_1 &= 4 \\ \hline 2(y_1 + y_2 + y_3) &= 10 \end{aligned}$$

$y_1 + y_2 + y_3 = 5$

$y_1 = (y_1 + y_2 + y_3) - (y_2 + y_3) = (5) - (8) = 5 - 8 = \underline{\underline{-3}}$

$y_2 = (y_1 + y_2 + y_3) - (y_1 + y_3) = (5) - (4) = 5 - 4 = \underline{\underline{1}}$

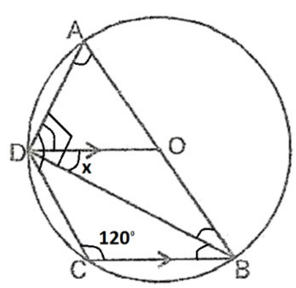
$y_3 = (y_1 + y_2 + y_3) - (y_1 + y_2) = (5) - (-2) = 5 + 2 = \underline{\underline{7}}$

$A(x_1, y_1) = (1, -3)$

$B(x_2, y_2) = (3, 1)$

$C(x_3, y_3) = \underline{\underline{(-5, 7)}}$

Answer 10.



(a) Given:

i. AB is a diameter.

ii. DO ∥ CB

iii. $\angle DCB = 120°$.

To find :

i. ∠DAB ii. ∠DBA iii. ∠DBC iv. ∠ADC

Proof: Δ AOD is equilateral.

Statement		Reason
1.	∠DCB = 120°	Given.
2.	∴ ∠DAB = 180° – 120° = 60°	Opposite angles of a cyclic quadrilateral are supplementary.
3.	∠ADB = 90°	Angle in a semi – circle is a right angle.
4.	∴ ∠ABD = 180°– (90° + 60°)= 30°	Sum of the angle of a triangle ABD is 180.
5.	In Δ ODB, ∠ODB = ∠OBD = 30°	OB and OD are radii of the same circle, Angles opposite equal sides are equal.
6.	∠DBC = ∠ODB = 30°	Interior alternate angles.
7.	In Δ DCB, ∠BDC = 180° – (120° + 30°) = 30°	 Sum of the angles of a triangle is 180°.
8.	∴ ∠ADC = 90° + 30° = 120°	By addition property.
9.	In Δ AOD, ∠OAD = 60° ∴ ∠ADO = ∠DAO = 60° ∠AOD = 180° – (60° + 60°) = 60°	From statement (2) above. AO and DO are radii of the same circle, Angles opposite equal sides are equal. Sum of the angles of a triangle is 180°.
10.	∴ Δ AOD is a equilateral triangle.	From statement 9 a, b and c.

(b) $\frac{\cos(90° - \theta)\cos\theta}{\cot\theta} = 1 - \cos^2\theta$

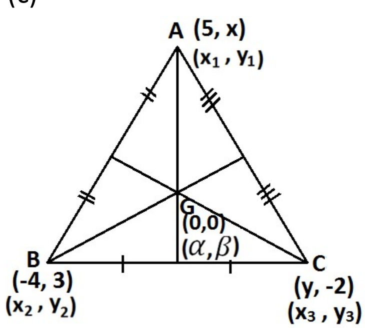
L.H.S $= \frac{\cos(90° - \theta)\cos\theta}{\cot\theta} = \frac{\sin\theta . \cos\theta}{\frac{\sin\theta}{\cos\theta}} = \sin\theta \times \cos\theta \times \frac{\sin\theta}{\cos\theta}$

$= \sin^2\theta = 1 - \cos^2\theta$

L.H.S = R.H.S

Hence proved

(c)



$\alpha = \frac{x_1 + x_2 + x_3}{3}$; $\beta = \frac{y_1 + y_2 + y_3}{3}$

$0 = \frac{5 - 4 + y}{3}$; $0 = \frac{x + 3 - 2}{3}$

$0 = 1 + y$; $0 = x + 1$

$y = 0 - 1$; $x = 0 - 1$

$y = -1$; $x = -1$

∴ $x = \underline{\underline{-1}}$

$y = \underline{\underline{-1}}$

Answer 11.

(a) Applying componendo & dividend,

$$\frac{3x + \sqrt{9x^2 - 5} + (3x - \sqrt{9x^2 - 5})}{3x + \sqrt{9x^2 - 5} - (3x - \sqrt{9x^2 - 5})} = \frac{5 + 1}{5 - 1}$$

$$\frac{3x + \sqrt{9x^2 - 5} + 3x - \sqrt{9x^2 - 5}}{3x + \sqrt{9x^2 - 5} - 3x - \sqrt{9x^2 - 5}} = \frac{6}{4}$$

$$\frac{6x}{2\sqrt{9x^2-5}} = \frac{3}{2}$$

Squaring both sides,

$$\frac{9x^2}{9x^2 - 5} = \frac{9}{4}$$

$$36\,x^2 = 81\,x^2 - 45$$

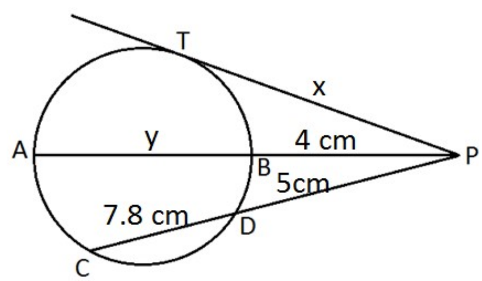
$$45 = 45\,x^2$$

$$x^2 = 1$$

$$x = \pm 1$$

(b) Given:

i. AB is the diameter.
ii. PT is a tangent.
iii. CD = 7.8 cm
iv. PD = 5cm
v. PB = 4 cm



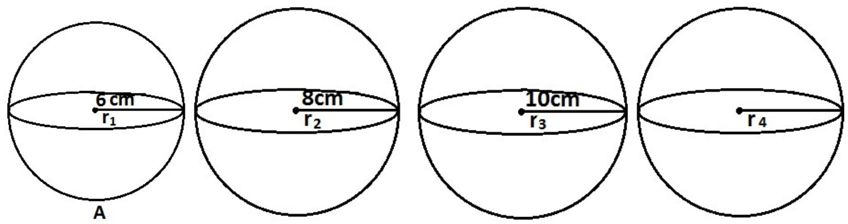
To find:

i. AB
ii. The length of tangent PT.

Statement		Reason
1.	Let PT = x cm	
2.	CD = 7.8 cm	Given.
3.	PD = 5 cm	Given.
4.	PC = PD + CD = 7.8 + 5 = 12.8 cm	By addition property.
5.	PT^2 = PC × PD = 12.8 × 5 = 64 = $\sqrt{64}$ = ± 8 = 8 cm	If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.
6.	Let AB = y cm	
7.	PB = 4 cm	Given.
8.	PA = PB + AB = 4 + y	By addition property.

9.	$PT^2 = PA \times AB$ $8^2 = (4 + y) \times 4$ $64 = 16 + 4y$ $4y = 48$ $y = \frac{48}{4} = \underline{\underline{12 \text{ cm}}}$	If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.

(c)



Volume of A + B + C = Volume of D

$\frac{4}{3}\pi(r_1)^3 + \frac{4}{3}\pi(r_2)^3 + \frac{4}{3}\pi(r_4)^3 = \frac{4}{3}\pi(r_4)^3$

$\frac{4}{3} \times \frac{22}{7}(6 + 8 + 10) = \frac{4}{3} \times \pi \times (r_4)^3$

$(r_4)^3 = 6^3 + 8^3 + 10^3$

$(r_4)^3 = 216 + 512 + 1000$

$(r_4)^3 = 1728$

$(r_4) = \sqrt[3]{1728}$

$(r_4) = 12 \text{ cm}$

Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 12 \times 12$

$= \frac{17856}{10}$

$= \underline{\underline{1785.6 \text{ cm}^2}}$

Answers of Practice Paper 10

Section I

Answer 1.

(a) $3x^2 + 5x - 9 = 0$

$a = 3, b = 5, c = -9$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{(5)^2-4(3)(-9)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{25+108}}{6}$

$x = \frac{-5 \pm \sqrt{133}}{6}$

$x = \frac{-5 \pm 11.533}{6}$

$x = \frac{-5 \pm 11.533}{6}$ or $x = \frac{-5 - 11.533}{6}$

$x = 1.088$ or $x = -2.755$

$x = 1.09$ or $x = - 2.76$

SS (x) = {1.09, -2.76}

(b) LHS $= \frac{1}{1-sin\theta} + \frac{1}{1+sin\theta} = \frac{1+sin\,\theta+1-sin\,\theta}{(1-sin\,\theta)(1+sin\,\theta)} = \frac{2}{(1-si\ \ ^2\,\theta)}$

$= \frac{2}{1-(1-c\ \ ^2\,\theta)}$ $(\because sin^2\,\theta + cos^2\,\theta = 1)$

$= \frac{2}{cos^2\,\theta} = 2sec^2\,\theta$ = RHS; hence proved.

(c) In ΔABC, $\tan 30^0 = \frac{AB}{BC}$; BC = ($\sqrt{3}$, x) m

In ΔBCF, $\tan 60^0 = \frac{BF}{BC}$; BC = ($\frac{50+}{\sqrt{3}}$) m

BC = BC

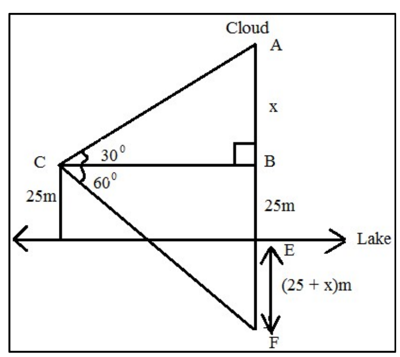
$\sqrt{3}x = (\frac{50+X}{\sqrt{3}})$

$3x = 50 + x$

x = 25m

∴ The cloud is = x+ 25

= 50m above the level.



Answer 2.

(a) A = $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$, C = $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$, D = $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

To find: AB + 2c – 4D

$= \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} -4 \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $= \begin{bmatrix} (3X6) & (-2X1) \\ (-1X6) & +(4X1) \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} -8 \\ 8 \end{bmatrix}$

$= \begin{bmatrix} 18 & -2 \\ -6 & +4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix}$ $= \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix}$

$= \begin{bmatrix} 16 - 8 \ - 8 \\ -2 + 10 + (-8) \end{bmatrix}$ $= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) x = Rs. 200,

n = $3\frac{1}{2}$ years

= 42 months

MV = Rs. 10,206

r = ?

MV = nx + $\left[\frac{nxr\ (n+1)}{2400}\right]$

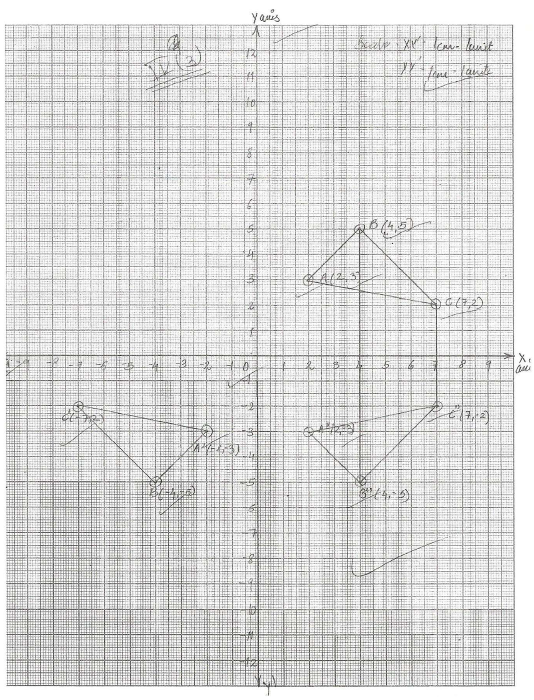
MV = (42 X 200) + $\left[\frac{42X200XrX4}{2400}\right]$

MV = 8400 + $\left(\frac{7X43Xr}{2}\right)$

$\frac{1806\ X\ 2}{7\ X\ 43}$ = r,

r = 12%

(c)



i. A (2, 3) $\xrightarrow{M_0}$ A' (-2,-3)
B (4, 5) $\xrightarrow{M_0}$ B' (-4,-5)
C (7, 2) $\xrightarrow{M_0}$ c'(-7,-2)

ii. A (2,+3) $\xrightarrow{M_x}$ A'' (2,-3)
B (4, 5) $\xrightarrow{M_x}$ B'' (4,-5)
C (7, 2) $\xrightarrow{M_x}$ C'' (7, -2)

iii. Isosceles trapezium

Area = $\frac{1}{2}$ (a + b) X h = $\frac{1}{2}$ X (4 + 10) X 3

Area = $\frac{1}{2}$ X 14 X 3 = 21 square unit

Answer 3.

(a) $\frac{x^3 + 12x + 6x^2 + 8}{x^3 + 12x - 6x^2 - 8} = \frac{y^3 + 27y + 9y^2 + 27}{y^3 + 27y - 9y^2 - 27}$

$\frac{y^3 + 6x^2 + 12x + 8}{y^3 - 6x^2 + 12x - 8} = \frac{y^3 + 9y^2 + 27y + 27}{y^3 - 9y^2 + 27y - 27}$

$\frac{(x+2)^3}{(x-2)^3} = \frac{(y+2)^3}{(y-2)^3}$

$\frac{x+2}{x-2}$	$= \frac{y+3}{y-3}$
$\frac{x+2+x-2}{x+2-x+2}$	$= \frac{y+3+y-3}{y+3-y+3}$
$\frac{2x}{4}$	$= \frac{2y}{6}$
$\frac{x}{2}$	$= \frac{y}{3}$
$\frac{x}{y}$	$= \frac{2}{3}$
x : y	= 2 : 3

(b) {No of possible events (n)} = 25

i. No of favourable events (m) = {2,4,6.8,10,12,14,16,18,20,22,24} = 12

∴ Probability of getting even [p (E)] = $\frac{m}{n} = \frac{12}{25}$

ii. No of favourable events (m) = {2, 3, 5, 7, 11, 13, 17, 19, 23} = 9

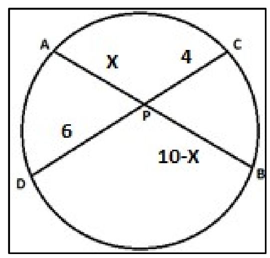
∴Probability of getting prime [p (E)] = $\frac{m}{n} = \frac{9}{25}$

iii. Multiple of 4

No of favourable events (m) = {4, 8, 12, 16, 20, 24} = 6

∴ Probability of getting multiple of 4 [p (E)] = $\frac{m}{n} = \frac{6}{25}$

(c)



i. Given : -

1. AB = 10 units,
2. DP = 6 units,
3. CP = 4 units

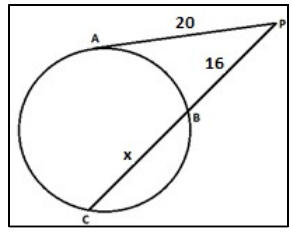
To find : -

1. AP

Statement	Reason
1. AB = 10 units	Given
2. AP = x unit	Given
3. DP = 6 units	Given
4. PC = 4 units	Given
5. BP = (10 – x) units	Addition property
6. AP × PB = PD × PC x × (10 – x) = 6 × 4 $10x - x^2$ = 24 0 = $x^2 - 10x + 24$ 0 = x(x – 6) -4 (x – 6) 0 = (x – 4) (x – 6) x – 4 = 0 ; x – 6 = 0 x = 4 units or x = 6 units AP = 4 or 6 units	When 2 chords intersect internally, the products of lengths of their segment are equal.

ii. Given : -

1. AP = 20 units,
2. PB = 16 units



To find : -

1. BC

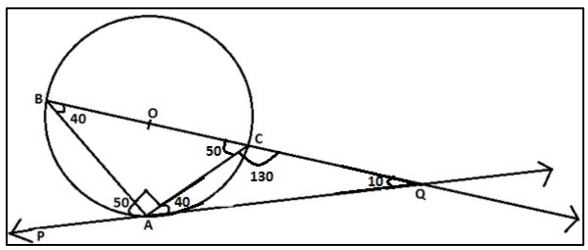
Statement	Reason
1. AP = 20 units	Given
2. PB = 16 units	Given
3. BC = x units	Given
4. PC = (16 + x) units	Addition property
5. $(AP)^2$ = PB X BCP $(20)^2$ = 16 X (16 + x) 400 = 256 + 16x 144 = 16x ∴ BC = 9 units	When a chord and a tangent intersect internally, the product of lengths of segments of the chord is equal to the square of length of tangent.

Answer 4.

(a) Given : -

i. ∠PAB = 50^0

ii. ∠ABC = 40^0



To find : -

i. ∠ACQ

ii. ∠AQB

Statement	Reason
1. PAB = 50°	Given
2. ∠ACB = ∠BAP = 50°	Alternate segment theorem
3. BAC = 90°	Angle in a semi – circle is a right angle.
4. ∠ABC = 40°	Given.
5. ∠ACB + ∠ACQ = 180° ∴ ∠ACQ = 180° – 50° = 130^0	Angles on a straight line sum upto 180°.
6. ∠CAQ = ∠ABC = 40°	
7. In Δ ACQ, ∠CAQ + ∠ACQ + ∠AQC = 180° 130° + 40° + ∠AQC = 180° 170° + ∠AQC = 180° ∠AQC = 180° – 170° ∠AQC = 10^0	Alternate segment theorem Sum of all ∠'s in a Δ = 180°

i. ∠ACQ = 130^0 ii. ∠AQB = 10^0

(b) Let $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ = k

Thus, x = ak, y = bk, z = ck

LHS:

$$= \frac{x^3}{a^3} - \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{(ak)^3}{a^3} - \frac{(bk)^3}{b^3} + \frac{(ck)^3}{c^3}$$

$$= \frac{a^3k^3}{a^3} - \frac{b^3k^3}{b^3} + \frac{c^3k^3}{c^3} = k^3 - k^3 + k^3$$

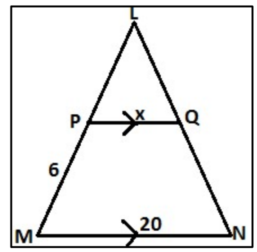
$$= \underline{\underline{k^3}}$$

RHS:

$$= \frac{xyz}{abc} = \frac{ak.bk.ck}{abc} = \frac{abck^3}{abc}$$

$$= \underline{\underline{k^3}}$$

Thus, <u>LHS = RHS.</u>



(c) <u>Given</u> : -

i. ∠P = 2 cm ii. PM = 6 cm iii. PQ ∥ MN iv. MN = 20 cm

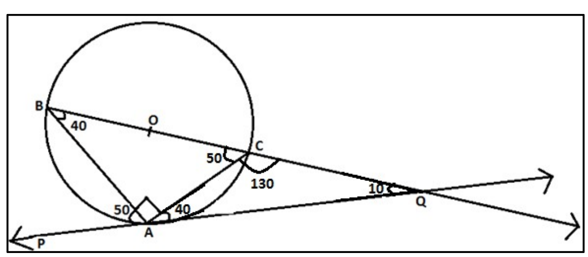
<u>To find</u> : -

i. PQ ii. A (trapezium PQNM): A (Δ LPQ)

Statement	Reason
1. In ΔLPQ and ΔLMN,	Common angle
∠PLQ = ∠LMN	Corresponding angles
∠LPQ = ∠LMN	By AA postulate
∴ ΔLPQ ∼ ΔLMN	Corresponding parts of similar triangles
2. $\frac{LP}{LM} = \frac{PQ}{MN} = \frac{LQ}{LN}$	
$\frac{2}{2+6} = \frac{x}{20}$	
$x = \frac{2 \times 20}{8}$	
x = <u>5</u>	
Thus <u>PQ = 5cm</u>	
3. $\frac{A\ trapezium\ PQNM}{A\ \Delta\ LPQ} = \frac{A\ \Delta LMN - A\ \Delta LPQ}{A \Delta LPQ}$	
$= \frac{(20)^2 - (5)^2}{(5)^2} = \frac{400-}{25}$	
$= \frac{375}{25} = \frac{15}{1} =$ <u>15 : 1</u>	

Section II

Answer 5.



(a) $$\frac{(4a+3)}{(9a+10)} = \frac{(3)^3}{(4)^3}$$

$$\frac{4a+3}{9a+10} = \frac{27}{64}$$

256a + 192 = 243a + 270

$13a = 78$

$a = \frac{78}{13} = \underline{6}$

(b) In Δ ABC, $(AC^2) = (BC^2) + (AB^2)$

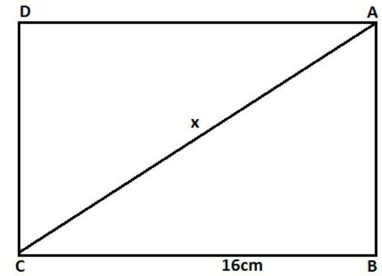
$(AC^2) = (12^2) + (16^2)$

$AC = \sqrt{144 + 256}$

$AC = \sqrt{400}$

$\underline{AC} = \underline{20\ cm}$

$K = \frac{1}{250000}$



K X actual dig. Dist = diag .dist, on map

$\frac{1}{250000} \times x = 20$

$x = 20 \times 250000$

$x = 5000000\ cm$

$x = 5000000 \times \frac{1}{100} \times \frac{1}{100} = \underline{50\ km}$

Area on map $= 16 \times 12 = 192\ cm^2$

Area on plot:-

K^2 X actual = map

$(\frac{1}{250000})^2$ X actual $= 192cm^2$

actual $= 192 \text{ X } (2,50,000)^2$

actual $= \frac{192 X 250000 X}{100 X 100 X 1000}$

actual $= 192 \text{ X } 6.25$

actual $= \underline{1200\ km^2}$

(c) n = 200

FV = Rs. 100

MV = Rs. 12

r = 15%

D of Manu $= \frac{\text{FVnr}}{100} = \frac{100 \times 200 \times 15}{100} =$ Rs. 3000

D of Sohan $= \frac{\text{Fvnr}}{100} = \frac{100 \times 200 \times 15}{100} =$ Rs 3000

Manu:- $\frac{3000}{24000} \times 100 = \frac{300}{24} = \underline{12.5\%}$

Sohan:- $\frac{3000}{20000} \times 100 = \frac{300}{20} = \underline{15\%}$

Difference in annual income = 0

Difference in rate of interest = 15 – 12.5 = 2.5%

Answer 6.

(a) $-2 < \frac{5x}{3} + 3 \leq \frac{x}{2} + 5\frac{1}{3}$; $x \in I$

$-2 < \frac{5x}{3} + 3$; $\frac{5x}{3} + 3 \leq \frac{x}{2} + 5\frac{1}{3}$

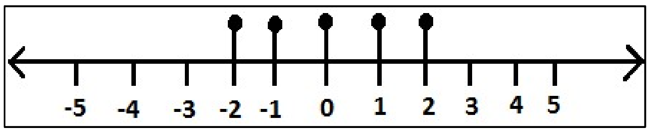
$-6 < 5x + 9$; $5x + 3 \leq \frac{3x+3}{6}$

$-6 < 5x + 9$; $18 + 10x \leq 3x + 32$

$-15 < 5x$; $+ x \leq 14$

$-3 < x \leq 2$

∴ S.S. (x) = {-2, -1, 0, 1, 2}



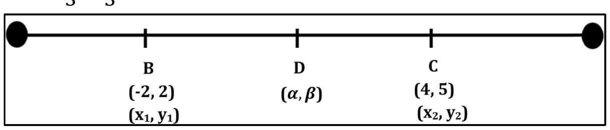
(b) Centroid G - (α, β)

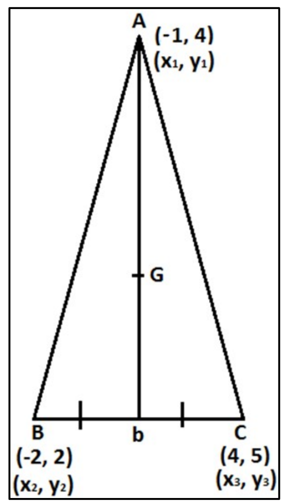
$\alpha = \frac{x_1 + x_2 + x_3}{3}$; $\beta = \frac{y_1 + y_2 + y_3}{3}$

$\alpha = \frac{-1-2+4}{3}$; $\beta = \frac{4+2+5}{3}$

$\alpha = \frac{1}{3}$; $\beta = \frac{11}{3}$

∴ G $(\alpha, \beta) = (\frac{1}{3}, \frac{11}{3})$





$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$\alpha = \frac{-2+4}{2}$; $\beta = \frac{2+5}{2}$

$\alpha = 1$; $\beta = \frac{7}{2} = 3\frac{1}{2}$

D $(\alpha, \beta) = (1, 3\frac{1}{2})$

Length of AD: A = (-1, 4) = (x_1, y_1)

D = $(1, \frac{7}{2})$ = (x_2, y_2)

AD = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}^2$

AD = $\sqrt{(1-1)^2 + (\frac{7}{2} - 4)^2}$

AD = $\sqrt{2^2 + \left(\frac{7-8}{2}\right)^2}$ = $\sqrt{4 + \frac{1}{4}}$ = $\sqrt{\frac{16+1}{4}}$

= $\sqrt{\frac{17}{4}}$ = $\sqrt{\frac{17}{2}}$ = $\frac{4.123}{2}$ = 2.06 units

(c)

Marks	F	fx	Cf
5	8	40	8
7	4	28	12
9	7	63	19
10	3	30	22
12	9	108	31
15	7	105	38
17	4	68	42
19	2	38	44
Total	44	480	

∴ Mean $= \frac{\sum fx}{\sum f}$

$= \frac{480}{44}$

Mean = 10.91 marks

Me = x value of $\left[\frac{n}{2}\right]^{th}$ term

= x value of 22^{nd} term

<u>Me</u> = <u>10 marks</u>

Mo = x value of highest frequency

Mo = x value of 9

<u>Mo</u> = <u>12 marks</u>

Answer 7.

(a) $3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

$f(x) = 3x^3 - kx^2 + 21x - 10$

$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 - k\left(\frac{2}{3}\right)^2 + 21\left(\frac{2}{3}\right) - 10$

$f\left(\frac{2}{3}\right) = \frac{8}{9} - \frac{4k}{9} + 14 - 10$

$f\left(\frac{2}{3}\right) = \frac{8 - 4k + 126 - 90}{9}$

Since 3x – 2 is a factor, the reminder is zero.

$0 = \frac{8 - 4k + 36}{9}$

$0 = -4k + 44$

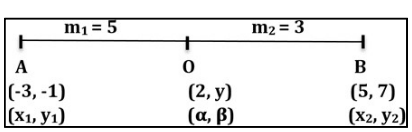
$4k = 44$

<u>K</u> = <u>11</u>

$$\begin{array}{r|l} & x^2 - 3x + 5 \\ \hline 3x - 2 & 3x^3 - 11x^2 + 21x - 10 \\ & \underline{-3x^3 \mp 2x^2} \\ & \quad -9x^2 + 21x \\ & \quad \underline{\mp 9x^2 - 6x} \\ & \qquad\quad 15x - 10 \\ & \qquad\quad \underline{+15x \mp 10} \\ & \qquad\qquad\quad 0 \end{array}$$

$3x^3 - 11x^2 + 21x - 10 = \underline{(3x - 2)(x^2 - 3x + 5)}$

(b)



$\alpha = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$\alpha = \frac{m_1 + (5) + (-3m_2)}{m_1 + m_2}$

$2m_1 + 2m_2 = 5m_1 - 3m_2$

$5m_2 = 3m_1$

$\frac{m_1}{m_2} = \frac{5}{3}$

$\beta = \frac{m_2\, y_1 + m_1\, y_2}{m_1 + m_2}$

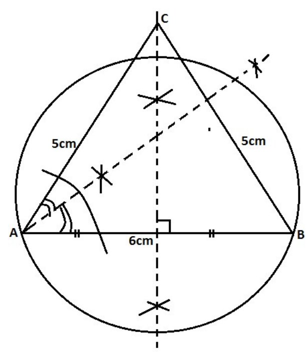
$y = \frac{5\,(7) + 3(-1)}{8}$

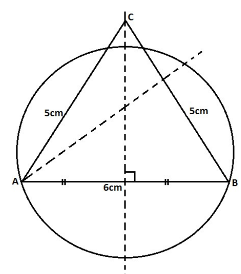
$y = \frac{35 - 3}{8}$

$y = 4$

A(2, y) = (2, 4)

(c)





PC = 2.3cm

Answer 8.

(a)

Car	Train
D = 216 km	D = 208 km
S = x km/hr	S = 208 $x + 16$ km/hr
T = $\left[\frac{216}{x}\right]$ hrs	T = $\left[\frac{208}{x+1}\right]$hrs

(i) Therefore time taken by car to reach the town B from A = $\left[\frac{216}{x}\right]$ hrs

(ii) Therefore time taken by train to reach the town B from A = $\left[\frac{208}{x+16}\right]$hrs

(iii) The equation is,

$\frac{216}{x} - \frac{208}{x+16} = 2$

$\frac{216}{x} = 2 + \frac{208}{x + 16}$

$\frac{216}{x} = \frac{2x + 32 + 208}{x + 16}$

$216x + 3456 = 2x^2 + 32x + 208$

$0 = 2x^2 + 24x - 3456$

0	=	$x^2 + 12x - 1728$
0	=	$x^2 + 48x - 36x - 1728$ = $x(x+48) - 36(x+48)$ = $(x-36)(x+48)$

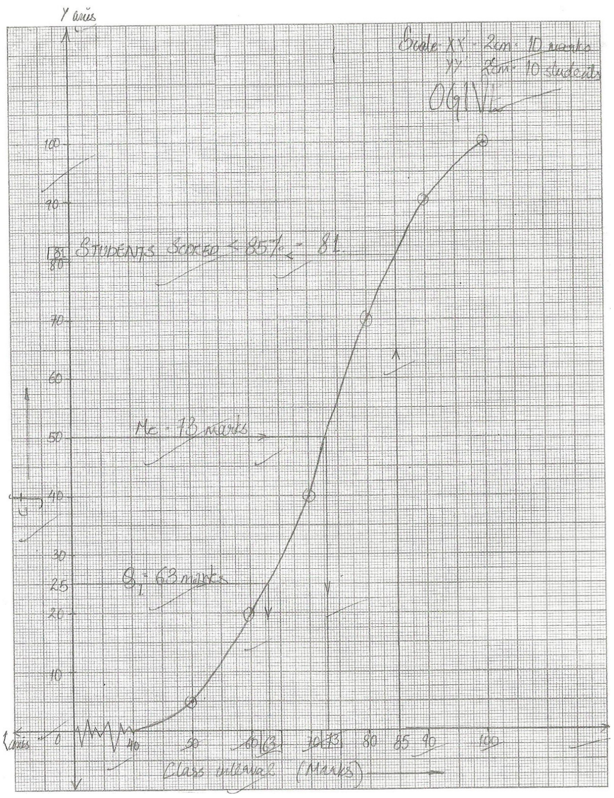
$$x = 36 \; or \; x = -48$$

Ignoring the – ve value

$$\underline{x = 36km/hr.}$$

(i)

(b)



Class interval (marks)	F	Cf
40-50	**5**	**5**
50-60	**15**	**20**
60-70	**20**	**40**
70-80	**30**	**70**
80-90	**20**	**90**
90-100	**10**	**100**
Total	**100**	

Me = x value of $[\frac{n}{2}]^{th}$ term = x value of 50^{th} term = <u>73 marks</u>

Q_1 = x value of $[\frac{n}{4}]^{th}$ term = x value of 25^{th} term = <u>63 marks</u>

no. of students getting > 85% = 100 – 81 = <u>19 students</u>

Answer 9.

(a) Ratio = (2x – 1) : (5x + 1)

Triplicate Ratio = 2 : 3

$$\frac{2x-1}{5x+1} = \frac{(2)^3}{(3)^3}$$

$$\frac{2x-1}{5x+1} = \frac{8}{27}$$

54x – 27 = 40x + 8

14x = 35

x = $\frac{35}{14}$ = $2\frac{7}{14}$ = $2\frac{1}{2}$ = <u>2.5</u>

(b) Let three numbers in A.P. be

a – d, a, a + d

∴ a – d + a + a + d = 15

$3a = 15$

$a = \frac{15}{3} = 5$

and $(a - d)^2 + (a + d)^2 = 58$

$a^2 + d^2 - 2ad + a^2 + d^2 + 2ad = 58$

$2(a^2 + d^2) = 58$

$a^2 + d^2 = \frac{58}{2} = 29$

$(5)^2 + d^2 = 29$

$25 + d^2 = 29$

$d^2 = 29 - 25 = 4$

$\therefore d = \pm 2$

Numbers will be 3, 5, 7 or 7, 5, 3.

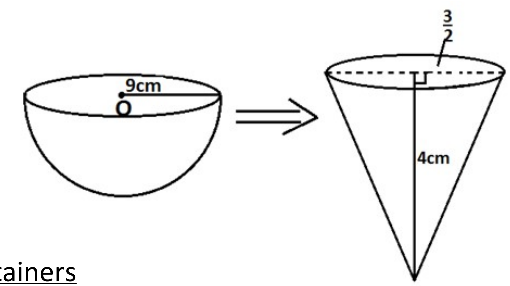
(c)

Volume of hemisphere = n × volume of cone

$n = \frac{\text{volume of hemisphere}}{\text{volume of cone}}$

$= \frac{\frac{2}{3} \times \pi \times 9^3}{\frac{1}{3} \times \pi \times \left(\frac{3}{2}\right)^2 \times 4}$

$= \frac{2 \times 9^3 \times 2^2}{3^2 \times 4}$ = <u>162 containers</u>



Answer 10.

(a) First term of a G.P.

(a) = 27

$T_8 = \frac{1}{81}$

$n = 10$

$a = 27,$

$ar^7 = \frac{1}{81}$ $\{T_n = ar^{n-1}\}$

$27r^7 = \frac{1}{81}$

$r^7 = \frac{1}{81} \times \frac{1}{27} = \frac{1}{3^4 \times 3^3} = \frac{1}{3^7}$

$r^7 = \left(\frac{1}{3}\right)^7$

$r = \frac{1}{3}$

Now $S_{10} = \frac{a\,(1 - r^n)}{1 - r}$ $(r < 1)$ $= \frac{27\left[1 - \left(\frac{1}{3}\right)^{10}\right]}{1 - \frac{1}{3}} = \frac{27\left(1 - \frac{1}{3^{10}}\right)}{\frac{2}{3}}$

$= \frac{27 \times 3}{2}\left(1 - \frac{1}{3^{10}}\right) = \frac{81}{2}\left(1 - \frac{1}{3^{10}}\right)$

(b)

i. Line PQ

$3y - 3x + 7 = 0$

$3y = 3x - 7$

$y = \frac{3x}{3} - \frac{7}{3}$

$y = x - \frac{7}{3}$

	y	$= mx + c$
	m	$= \underline{\underline{1}}$
ii.	m	$= \tan\theta$
	1	$= \tan\theta$
	θ	$= \underline{\underline{45^\circ}}$
iii.	y – intercept of the line	$= \frac{-7}{3}$

(c) $f(x) = 3x^3 + 10x^2 + x - 6$

Let $(x + 1)$ be the factor

$x + 1 = 0$

$x = -1$

$f(-1) = 0$

$f(x) = 3x^3 + 10x^2 + x - 6$

$f(-1) = 3(-1)^3 + 10(-1)^2 + (-1) - 6$

$0 = 3(-1) + 10(1) - 1 - 6$

$0 = -3 + 10 - 1 - 6$

$0 = -10 + 10$

$0 = 0$

Thus $(x + 1)$ is a factor.

$$
\begin{array}{r|l}
 & 3x^2 + 7x - 6 \\
\hline
x + 1 & 3x^3 + 10x^2 + x - 6 \\
 & \underline{3x^3 + 3x^2} \\
 & \quad (-) \quad (-) \\
 & 7x^2 + x \\
 & \underline{7x^2 + x} \\
 & \quad (-) \quad (-) \\
 & -6x - 6 \\
 & \underline{-6x - 6} \\
 & \quad (+) \quad (+) \\
 & 0
\end{array}
$$

$3x^3 + 10x^2 + x - 6 = (x + 1)(3x^2 + 7x - 6)$

$= (x + 1)(3x^2 + 9x - 2x - 6)$

$= (x + 1)[3x(x + 3) - 2(x + 3)]$

$= \underline{\underline{(x + 1)(x + 3)(3x - 2)}}$

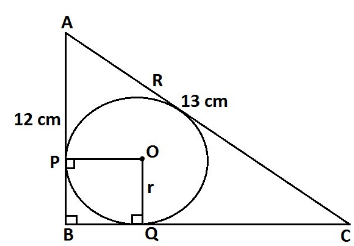
Answer 11.

(a) Given :- AB = 12cm,

AC = 13cm,

$\angle B = 90^\circ$

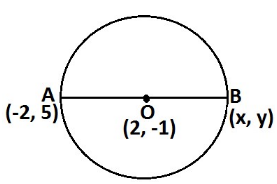
To find :- r

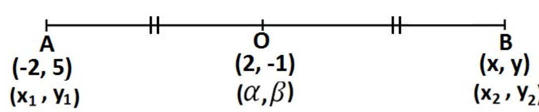


Statement	Reason
1) $BC = \sqrt{13^2 - 12^2}$ $= \sqrt{169 - 144}$ $= \sqrt{25}$ $= \underline{\underline{5}}$	By Pythagoras theorem.
2) BP = BQ	Tangents from a point outside the circle are equal.

3) OP = OQ = r	Radii of the same circle.
4) ∠OPB = ∠OQB = 90°	Tangents are ⊥ to circle.
5) ∠ABC = 90°	Given
6) ∠POQ = 360° - (90° + 90° + 90°) = 90°	Sum of all angles of the square is 360°.
7) ∴ BPOQ is a square.	Adjacent sides equal and all angles are 90°.
8) ∴ BP = BQ = OP = OQ = r	Equal sides of a square.
9) BC = 5cm	From statement 1 above.
10) QC = 5 – r	By addition property.
11) QC = CR = 5 – r	Tangents from a point outside the circle are equal.
12) AR = 13 - (5 – r) = 13 – 5 + r = 8 + r	By addition property.
13) AR = AP = 8 + r	Tangents from a point outside the circle are equal.
14) AB = AP + BP 12 = (8 + r) + r 12 -8 = 2r r =2	By addition property.

(b)





$$\alpha = \frac{x_1 + x_2}{2} \quad ; \quad \beta = \frac{y_1 + y_2}{2}$$

$$2 = \frac{-2 + x}{2} \quad ; \quad -1 = \frac{5 + y}{2}$$

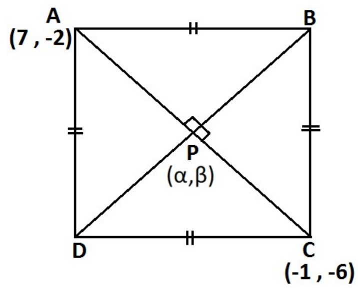
$$4 = -2 + x \quad ; \quad -2 = 5 + y$$

$$x = 4 + 2 \quad ; \quad y = -2 - 5$$

$$x = 6 \quad ; \quad y = -7$$

$$B(x, y) = \underline{\underline{(6, -7)}}$$

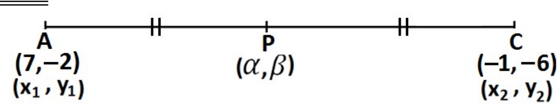
(c)



i. <u>Equation of diagonal AC:</u>

A	=	$(7, -2) = (x_1, y_1)$
C	=	$(-1, -6) = (x_2, y_2)$
$\frac{y - y_1}{x - x_1}$	=	$\frac{y_2 - y_1}{x_2 - x_1}$
$\frac{y - (-2)}{x - (7)}$	=	$\frac{(-6) - (-2)}{(-1) - (7)}$
$\frac{y + 2}{x - 7}$	=	$\frac{-6 + 2}{-8}$
$\frac{y + 2}{x - 7}$	=	$\frac{-4}{-8}$
$\frac{y + 2}{x - 7}$	=	$+\frac{1}{2}$
$2y + 4$	=	$+x - 7$
$-x + 2y$	=	$-7 - 4$
$-x + 2y$	=	-11
$\underline{\underline{x - 2y}}$	=	$\underline{\underline{11}}$

ii. <u>Coordinate of P:</u>



$\therefore \alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$\alpha = \frac{(7) + (-1)}{2}$; $\beta = \frac{(-2) + (-6)}{2}$

$\alpha = \frac{6}{2}$; $\beta = \frac{-2 - 6}{2}$

$\alpha = \underline{\underline{3}}$; $\beta = \frac{-8}{2}$

; $\beta = \underline{\underline{-4}}$

<u>P $(\alpha, \beta) = (3, -4)$</u>

iii. $\underline{\underline{\therefore \text{m of AC}}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-6) - (-2)}{(-1) - (7)} = \frac{-6 + 2}{-1 - 7}$

$= \frac{-4}{-8} = \frac{1}{2}$

$\therefore$ $\underline{\underline{\text{m' of BD}}} = -\left(\frac{1}{m\ of\ AC}\right)$, perpendicular $= -\frac{1}{\frac{1}{2}} = \underline{\underline{-2}}$

∴ Equation of diagonal BD

$$m = -2$$

$$P = (3, -4) = (x_1, y_1)$$

$$m = \frac{y - y_1}{x - x_1}$$

$$-2 = \frac{(y) - (3)}{(x) - (-4)}$$

$$-2 = \frac{y - 3}{x + 4}$$

$$-2(x + 4) = y - 3$$

$$-2x - 8 = y - 3$$

$$-2x - y = 8 - 3$$

$$-2x - y = 5$$

$$2x + y = -5$$

$$2x + y = 2$$

Answers of Practice Paper 11

Section I

Answer 1.

(a) ∴ L.H.S

$$\sqrt{\frac{1-\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{1-\sin A}{1+\sin A} \times \frac{1+\sin A}{1+\sin A}} \qquad = \sqrt{\frac{1-sin^2 A}{1+sin^2 A}} \qquad = \frac{\sqrt{1-sin^2 A}}{1+\sin A}$$

$$= \frac{\sqrt{cos^2 A}}{1+\sin A} \qquad = \frac{cos A}{1+sin A}$$

R.H.S

$$= \frac{cos A}{1+sin A}$$

<u>L.H.S = R.H.S</u>

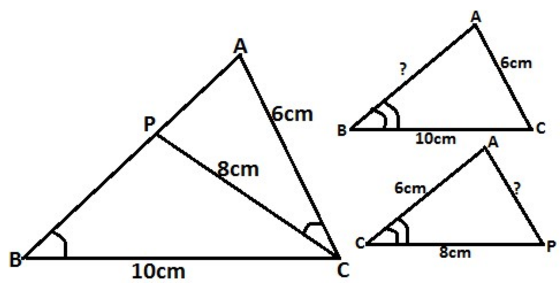
<u>Hence proved.</u>

(b) Given:

i. AC = 6 cm ii. PC = 8 cm iii. BC = 10 cm

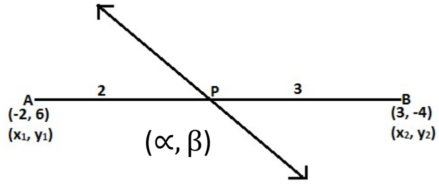
To find:

i. AB ii. AP iii. $\frac{Area\ \Delta ABC}{Area\ \Delta APC}$



Statement.	Reason.
1. In ΔABC and ΔACP, ∠A = ∠A ∠ABC = ∠ACP thus, ΔABC ≅ ΔACP	 Common Angle. Given in the diagram. By AA test of similarity.
2. ∴ $\frac{AB}{AC} = \frac{AC}{AP} = \frac{BC}{PC}$, Thus, $\frac{AC}{AP} = \frac{BC}{PC}$ ∴ $\frac{6}{AP} = \frac{10}{8}$, <u>AP = 4.8cm</u>	Corresponding sides of Similar triangles are similar (BPT).
3. ∴ $\frac{AB}{AC} = \frac{BC}{PC}$, Thus, ∴ $\frac{AB}{6} = \frac{10}{8}$ AB = <u>7.5cm</u>	Corresponding sides of Similar triangles are similar (BPT).
4. $\frac{A\,\Delta\,(ABC)}{A\,\Delta\,(APC)} = \frac{BC^2}{PC^2}$ $= \frac{(10)^2}{(8)^2}$ $= \frac{100}{64}$ $= \frac{25}{16}$ A ΔABC : AΔAPC = <u>25 : 16</u>	Ratio of areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

(c)



$\propto = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$, $\beta = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\propto = \frac{2(3) + 3(-2)}{2+3}$, $\beta = \frac{2(-4) + 3(6)}{2+3}$

$\propto = \frac{6-6}{5}$, $\beta = \frac{-8+18}{5}$

$\propto = \frac{0}{5} = 0$, $\beta = \frac{10}{5} = 2$

P = {∝, β} = {0, 2}

m $= \frac{3}{2}$

P = {0, 2} = x_1, y_1

$m(x - x_1) = y - y_1$

$\frac{3}{2}(x - 0) = y - 2$

3x = 2y – 4

<u>3x – 2y = -4</u>

Answer 2.

(a) Total outcomes = {1, 2, 3....20}

Total no. of outcomes = 20.

i. Favourable outcome = a prime no. {2, 3, 5, 7, 11, 17, 19}

i) Favourable outcome = 8

P(a prime no.) $= \frac{\text{No.of avourable outcomes}}{\text{total no.of outcomes.}}$

P(a prime no.) $= \frac{8}{20}$

P(a prime no.) $= \frac{1}{4}$

i. Favourable outcome = a number divisible by 3 {3, 6, 9, 12, 15, 18}

ii) Favourable outcome = 6

P(a no. divisible by 3) $= \frac{\text{No.of favourable outcomes}}{\text{total no.of outcomes}}$

P(a no. divisible by 3) $= \frac{6}{20}$

P(a no. divisible by 3) $= \frac{3}{10}$

i. Favourable outcome = a perfect square {1, 4, 9, 16}

iii) Favourable outcome = 4

P(a perfect square) $= \frac{\text{No.of favourable outcomes}}{\text{total no.of outcomes}}$

P(a perfect square) $= \frac{5}{20}$

P(a perfect square) $= \frac{1}{4}$

(b) ∴ L.H.S

$$= \frac{1}{\sin A + \cos A} + \frac{1}{\sin A - \cos A} = \frac{\sin A - \cos A + \sin A + \cos A}{(\sin A + \cos A)(\sin A - \cos A)}$$

$$= \frac{2 \sin A}{\sin^2 A - \cos^2 A} = \frac{2 \sin A}{1 - \cos^2 A - \cos^2 A} = \frac{2 \sin A}{1 - 2\cos^2 A}$$

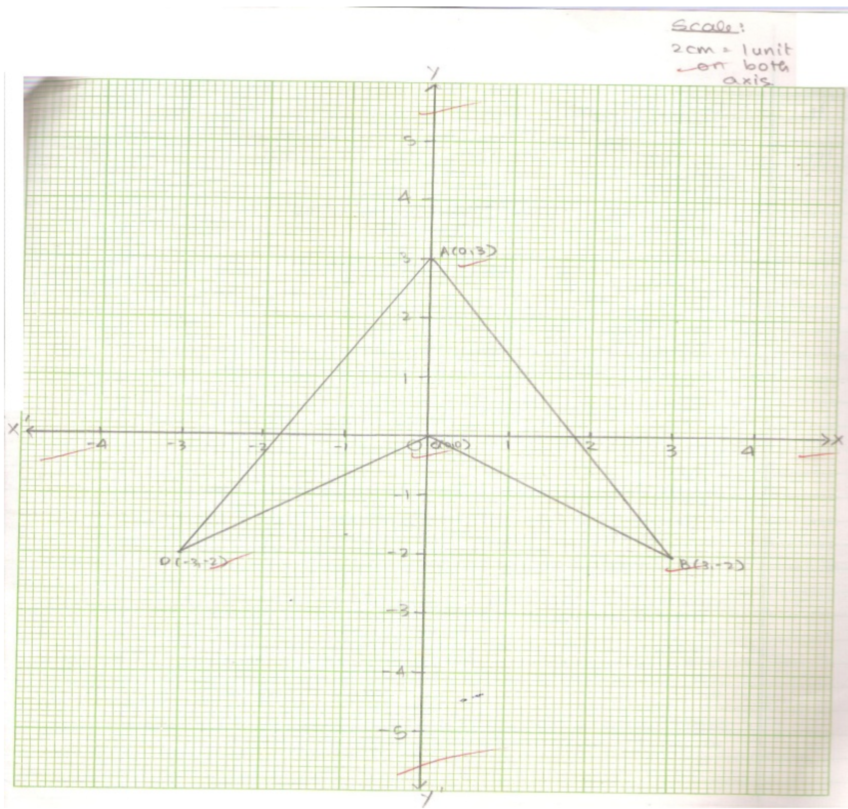
R.H.S

$$= \frac{2 \sin A}{1 - 2\cos^2 A}$$

<u>L.H.S = R.H.S</u>

<u>Hence proved</u>

(c)



Answer 3.

(a)

∴ Volume of cone $= \frac{2}{3}$ volume of hemisphere

$\frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{2}{3}\pi r^3$

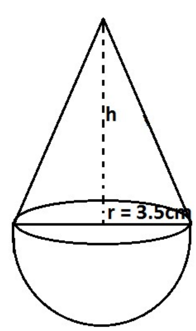
$\frac{1}{3} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times h = \frac{4}{9} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times \frac{35}{10}$

$\frac{1}{3} h = \frac{4}{9} \times \frac{35}{10}$

$h = \frac{4 \times 35 \times 3}{9 \times 10}$

$h = \frac{14}{3}$

h = <u>4.66 m</u>



Surface area buoy $= 3\pi r^2 = 3 \times \frac{22}{7} \times \frac{466}{100} \times \frac{466}{100}$

l $= \sqrt{(4.66)^2 + (3.5)^2}$ $= \sqrt{21.756 + 12.25}$

$= \sqrt{1246.76}$

Surface area of buy $= \pi r^2 + \pi r$ (r + l) $= \left(\frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}\right) + \frac{22}{7} \times \frac{35}{10} \times (3.5 + 5.84)$

= 38.5 + 11 (3.5 + 5.84) = 38.5 + 11 (9.34)

= 38.5 + 1.0274 = <u>141.13 m²</u>

(b) $\frac{(\sqrt{x+5} + \sqrt{x-16}) + (\sqrt{x+5} - \sqrt{x-16})}{(\sqrt{x+5} + \sqrt{x-16}) - (\sqrt{x+5} - \sqrt{x-16})} = \frac{7+3}{7-3}$ (By C & D)

$\therefore \frac{\sqrt{x+5} + \sqrt{x-16} + \sqrt{x+5} - \sqrt{x-16}}{\sqrt{x+5} + \sqrt{x-16} - \sqrt{x+5} + \sqrt{x-16}} = \frac{10}{4}$

$\frac{2\sqrt{x+5}}{2\sqrt{x-16}} = \frac{5}{2}$

$\frac{x+5}{x-16} = \frac{25}{4}$

4x + 20 = 25x – 400

420 = 21x

x = <u>20</u>

(c) Let the no. of articles be 'x'

Cost of x articles $= \frac{1200}{x}$

Damaged articles = 10

Left articles = x – 10

Thus, SP of one article $= \frac{1200+2x}{x}$

Total SP = CP × n

CP + P $= \frac{(1200 + 2x)(x-10)}{x}$

1260 $= \frac{(1200+ \quad)(x-10)}{x}$

1260x $= 2(1200 + 2x)(x - 10)$

630x = $600x - 6000 + x^2 - 10x$

0 = $x^2 - 630x + 600x - 10x - 6000$

0 = $x^2 - 40x - 6000$

0 = $x^2 - 100x + 60x - 6000$

0 = x(x – 100) + 60(x – 100)

0 = (x – 100)(x + 60)

Thus, x = 100 or -60

Ignoring the –ve sign we get: <u>x = 100 articles.</u>

Answer 4.

(a) $f(x) = ax^3 + 3x^2 - 13$

$f(-2) = a(-2)^3 + 3(-2)^2 - 13$

f(-2) = -8a + 12 – 13

f(-2) = -8a – 1

f(-2) = -8a - 1

$g(x) = 2x^3 - 5x + a$

$f(-2) = 2(-2)^3 - 5(-2) + a$

f(-2) = 2(-8) + 10 + a

f(-2) = -16 + 10 + a

f(-2) = -6 + a

since the remainders are same:

$-8a - 1 = -6 + a$

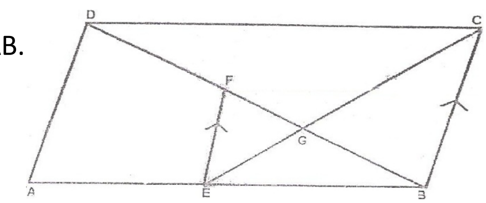
$-8a - a = -6 + 1$

$-9a = -5$

$9a = 5$

$a = \frac{5}{9}$

(b) Given:



i. ABCD is a parallelogram ii. E is a point on AB.

iii. CE intersects the diagonal BD at G

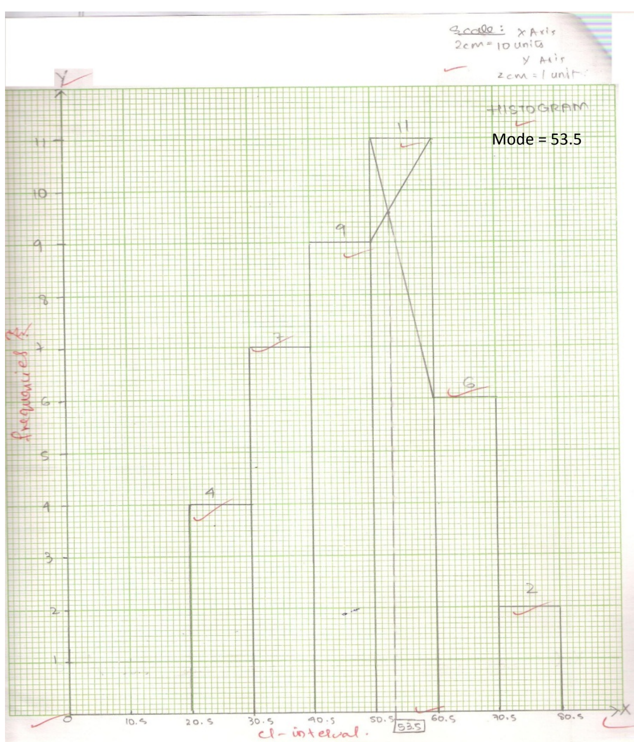
iv. EF is parallel to BC. v. AE: EB = 1 : 2,

To find:

i. EF : AD ii. Area of ΔBEF : Area of ΔABD

Statement.	Reason.
1. In ΔBEF and ΔBAD: ∠BFE = ∠BDA. ∠B = ∠B thus, ΔBEF ≅ ΔBAD	 Corresponding angles. Common∠. By AA postulate.
2. $\frac{BE}{BA} = \frac{BF}{BD} = \frac{EF}{AD}$ $\frac{2}{3} = \frac{BF}{BD} = \frac{EF}{AD}$ <u>EF : AD = 2 : 3</u>	Corresponding sides of similar triangles are similar.
3. $\frac{A(\Delta BEF)}{A(\Delta BAD)} = \frac{EF^2}{AD^2} = \frac{2^2}{3^2}$ A(ΔBEF) : A(ΔBAD) = 4 : 9	Ratio of area of 2 similar triangles is the ratio of square of their similar sides.

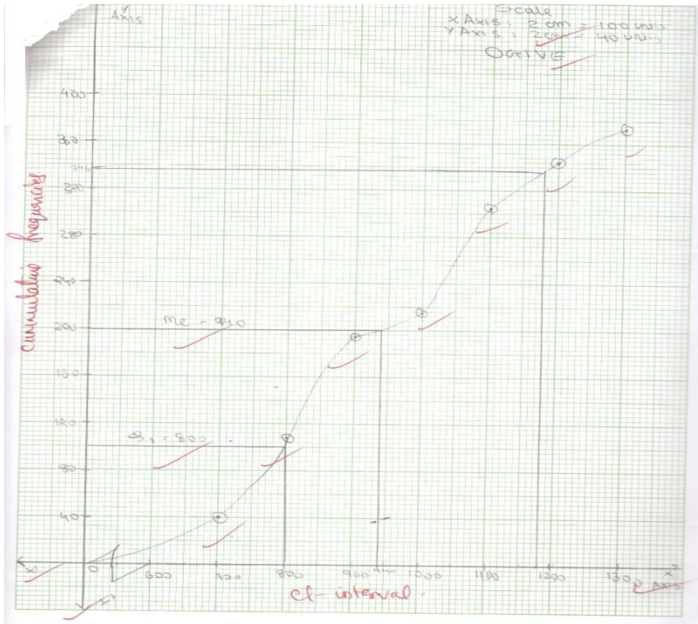
(c)



Section II

Answer 5.

(a)



(i) Median = Rs. 940

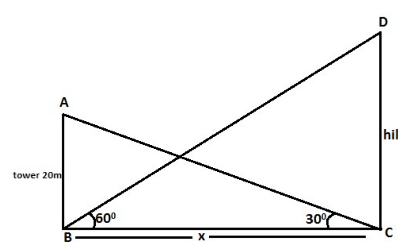
(ii) Number of Employees whose income exceeds Rs. 1180 = 400 – 346 = Rs. 54

(iii) Q_1 = Rs. 800

Q_3 = Rs. 1080

Therefore inter quartile range = $Q_3 - Q_1$ = 1080 – 800 = Rs. 280

(b)



$\tan 60° = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

$x = \frac{h}{\sqrt{3}}$ ------(i)

$\tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{20}{x}$

$x = 20\sqrt{3}$ ------(ii)

equating eq. (i) and (ii):

$\frac{h}{\sqrt{3}} = 20\sqrt{3}$

<u>h = 60m</u>

thus, <u>x = $20\sqrt{3}$</u>

Answer 6.

(a) $\frac{x^3 + 3x}{3x^2 + 1} = \frac{341}{91}$

By using componendo-dividendo:

$$\frac{x^3 + 3x + 3x^2 + 1}{x^3 + 3x - 3x^2 - 1} = \frac{341+}{341-}$$

$$\frac{x^3 + 3x(x+1) + 1}{x^3 + 3x(x-1) - 1} = \frac{432}{250}$$

$$\frac{(x+1)^3}{(x-1)^3} = \frac{216}{125}$$

Taking cube root on both the sides, we get:

$$\frac{x+1}{x-1} = \frac{6}{5}$$

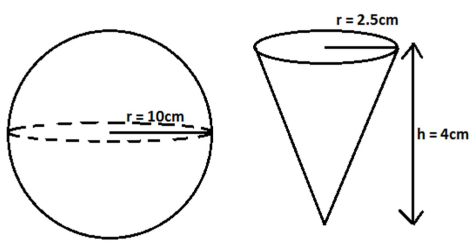
5(x + 1) = 6(x - 1)

5x + 5 = 6x – 6

-x = -11

x = <u>11</u>

(b)



Surface Area (SA) = $4\pi r^2$

SA = $4 \times \frac{22}{7} \times 10^2$

SA = $4 \times \frac{22}{7} \times 100$

SA = $\frac{8800}{7}$

SA = <u>1257.14cm^2</u>

V (of sphere) = n × V (of cone)

$\frac{4}{3}\pi r^3 = n \times \frac{1}{3}\pi r^2 h$

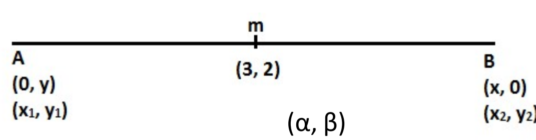
$\frac{4}{3}\pi(10 \times 10 \times 10) = n \times \frac{1}{3}\pi\left(\frac{25}{10} \times \frac{25}{10}\right)4$

$4000 = n \times \frac{625}{100} \times 4$

$n = \frac{4000 \times 100 \times 4}{625}$

n = <u>160</u>

(c)



i) $\alpha = \frac{x_1 + x_2}{2}$, $\beta = \frac{y_1 + y_2}{2}$

$3 = \frac{0+x}{6}$, $2 = \frac{y+0}{2}$

x = 18, y = 4

ii) A = (0, 4),   B(6, 0)

A = (0, 4) = (x_1, y_1)

B = (6, 0) = (x_2, y_2)

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

$\frac{0 - 4}{6 - 0} = \frac{y - 4}{x - 0}$

<u>3y + 2x = 12</u>

iii) m of AB = $\frac{-4}{6} = \frac{-2}{3}$ -----from (ii)

$m_1 \times m_2 = -1$

$\frac{-2}{3} \times m_2 = -1$

$m_2 = \frac{3}{2}$

Equation of OP: 0 = (0,0)

$= (x_1, y_1)$

m $= \frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{3}{2}(x - 0)$

-3x + 2y = 0

(iv) Equation of OP:

-3x + 2y = 0

2x + 3y = 12

Solve both the equation.

Therefore x = 1.8

y = 2.8.

Answer 7.

(a) x(2x – 7) = 3

$2x^2 - 7x = 3$

$2x^2 - 7x - 3 = 0$

a = 2, b = -7, c = -3

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$

$x = \frac{7 \pm \sqrt{49+24}}{4}$

$x = \frac{7 \pm \sqrt{73}}{4}$

$x = \frac{7 \pm 8.506}{4}$

$x = \frac{7 + 8.506}{4}$, $\quad x = \frac{7 - 8.506}{4}$

$x = \frac{15.506}{4}$, $\quad x = \frac{-1.506}{4}$

$x = 3.6265$, $\quad x = -0.378$

x = (3.63, -0.38) ----------(2 decimal)

x = (3.6. -0.28) -----------(2 significant numbers)

(b)

f(x)	$= 2x^3 + 3x^2 - kx + 5$
Divisor	= x – 2
Remainder	= 7
f (2)	= 7
f (2)	$= 2x^3 + 3x^3 - kx + 5$
7	$= 2\,(2)^3 + 3\,(2)^2 - k\,(2) + 5$
7	= 16 + 12 – 2k + 5
7	= 33 – 2k
2k	= 26
K	= 13

(c)

x = Rs. 400,

n = 15months,

r = 8%

MV = (nx) + $\left(\frac{nxr\,(n+1)}{2400}\right)$

MV = (6000) + $\left(\frac{15\ X\ 400X8\ X\ 16}{2400}\right)$

MV = 6000 + 320

<u>MV = Rs. 6320</u>

Answer 8.

(a)

I_1	I_2
FV_1 = Rs. 100	FV_2 = Rs 50
$n_1 = n$	$n_2 = 160 \times \frac{n}{48}$
r_1 = 15%	r_2 = 18%
MV_1 = –	MV_2 = Rs. 48
I_1 = –	I_2 = –
D_1 = –	D_2 = –
R_1 = –	R_2 = –
Sale MV = Rs. 160	
Sale P = –	

Increase in income = Rs. 900

Sale P = Sale MV × n

I_2 = 160 × n

$n_2 = \frac{I_2}{MV_2} = \frac{160n}{48}$ Shares

$D_2 = \frac{FV_2 \times n_2 \times r_2}{100} = \frac{50 \times 160 \times n \times 18}{48 \times 100} = \underline{\underline{30n}}$

$D_1 = \frac{FV_1 \times n_1 \times r_1}{100} = \frac{100 \times n \times 15}{100} = 15\,n$

Diff = $D_2 - D_1$

900 = 30n – 15n

900 = 15n

$n = \frac{900}{15} = \underline{\underline{\text{60 shares}}}$

(b) $4x - 19 \leq \frac{3x}{5} - 2$; $\frac{3x}{5} - 2 < x - \frac{1}{5}$

$4x - 19 \leq \frac{3x - 10}{5}$; $\frac{3x - 10}{5} < \frac{5x - 1}{5}$

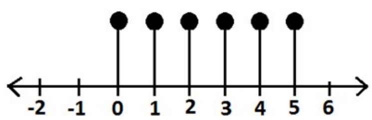
$20x - 95 \leq 3x - 10$; $3x - 10 < 5x - 1$

$-85 \leq -17x$; $-2x < 9$

$5 \geq x$; $x > -4.5$

$-4.5 < x \leq 5$

x = $\underline{\underline{\{0, 1, 2, 3, 4, 5\}}}$



(c) $SI = \frac{Pnr}{100}$

$SI = \frac{420\cancel{00} \times r \times 2}{1\cancel{00}}$

SI = 840r

$CI = P\left(1 + \frac{r}{100}\right)^2 - P$

$CI = 42000\left[\left(\frac{100+r}{100}\right)^2 - 1\right]$

$CI = 42000\left[\frac{\cancel{10000} + r^2 + 200r - \cancel{10000}}{10000}\right]$

$CI = 42\left[\frac{200r + r^2}{10}\right]$

Difference = CI – SI

$105 = 42\left[\frac{200r + r^2}{10}\right] - 840r$

$1050 = \cancel{8400r} + 42r^2 - \cancel{8400r}$

$1050 = 42r^2$

$\frac{1050}{42} = r^2$

$r = \sqrt{25}$

r = 5%

Answer 9.

(a) A total no. of orange boxes = 1077

Total no. of orange boxes rotten = 27

∴ Remaining boxes = 1077 – 27 = 1050

∴ Ratio of remaining boxes = 4: 5 : 6

∴ Let the common multiple = x

∴ Total no. of remaining boxes = 15x

15x = 1050

x = 70

∴ Remaining no. of boxes in: A = $4x$ = 70×4 = 280

∴ Original no. of loaded = 280 + 7 = 287

B = $5x$ = 5×70 = 350

C = 6x = 6×70 = 420

∴ Original no. of loaded = 420 + 12 = 432.

(b) $(a^2 + b^2)(x^2 + y^2) = (ax + by)^2$

$a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2 = a^2x^2 + 2axby + b^2y^2$

∴ $a^2y^2 + b^2x^2 = 2axby$

∴ $a^2y^2 - 2axby + b^2x^2 = 0$

∴ $(ay - bx)^2 = 0$

$ay - bx = 0$

$ay = bx$

$\frac{a}{x} = \frac{b}{y}$

Hence proved.

(c) $f(x) = 2x^3 + x^2 - 13x + 6$

∴ Let (x – 2) be the factor

∴ $x - 2 = 0$

$x = 2$

∴ $f(2) = 0$

$f(x) = 2x^3 + x^2 - 13x + 6$

$f(2) = 2(2)^3 + (2)^2 - 13(2) + 6$

$0 = 2(8) + 4 - 26 + 6$

$0 = 16 + 4 - 26 + 6$

0 = − 6 + 6

0 = 0

Thus (x – 2) is a factor.

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x-2 & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 - 4x^2 \\ & (-)\quad (+) \\ \hline & \quad 5x^2 - 13x \\ & \quad 5x^2 - 10x \\ & \quad (-)\quad (+) \\ \hline & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ & \qquad (+)\quad (-) \\ \hline & \qquad\qquad 0 \\ \hline \end{array}$$

$2x^2 + x^2 - 13x + 6 = (x - 2)(2x^2 + 5x - 3)$

$= (x - 2)(2x^2 + 6x - x - 3)$

$= (x - 2)[2x(x + 3) - 1(x + 3)]$

$= \underline{(x - 2)(x + 3)(2x - 1)}$

Answer 10.

(a) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$C = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

i. BC $= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix}$

A(BC) $= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 9 & 0 \end{bmatrix}$

ii. AB $= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

(AB)C $= \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 9 & 0 \end{bmatrix}$

iii. BC $= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix}$

(BC)A $= \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 3 & 0 \end{bmatrix}$

(b) 3 digits numbers are 100, 101, 102, ………., 999

Number when divided by 5, remainder = 3

$$\begin{array}{r|l} & 20 \\ \hline 5 & 103 \\ & \underline{100} \\ & \quad 3 \end{array} \qquad \begin{array}{r|l} & 199 \\ \hline 5 & 999 \\ & \underline{5} \\ & 49 \\ & \underline{45} \\ & \ 49 \\ & \ \underline{45} \\ & \quad 4 \end{array}$$

∴ 103, 108, 113, 118, ..., 998

First term (a) = 103

d = 108 – 103

= 5

l = 998

∴ l = a + (n – 1)d

998 = 103 + (n – 1) × 5

998 – 103 = (n – 1) × 5

$\frac{895}{5}$ = n – 1

179 = n – 1

n = 179 + 1 = 180

∴ S_n = $\frac{n}{2}$ [2a + (n – 1)d] = $\frac{180}{2}$ [2 × 103 + (180 – 1) × 5]

= 90 [206 + 179 × 5] = 90 [206 + 895]

= 90 × 1101 = 99090

(c) In a G.P.

Common ratio (r) = 3

Last term (l) = 486

Sum of its terms (S_n) = 728

Let a be the first term, then

l = ar^{n-1}

486 = ar^{n-1}

486 = $a(3)^{n-1}$

$a(3)^{n-1} = 486$ .... (i)

S_n = $\frac{a(r^n - 1)}{r - 1}$ (∵ r > 1)

728 = $\frac{a(3^n - 1)}{3 - 1}$

728 = $\frac{a(3^n - 1)}{2}$

728 × 2 = $a(3^n - 1)$

$a(3^n - 1)$ = 1456

$a \times 3^n - a$ = 1456

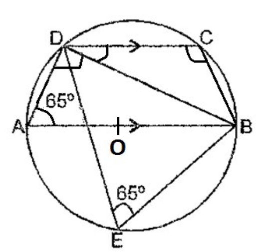
$a \times 3^{n-1} \times 3 - a$ = 1456

1458 – a = 1456

a = 1458 – 1456 = 2

Hence first term = 2

Answer 11.



(a) Given:

i. ∠BED = 65°

ii. AB ∥ CD

AB is a diameter.

To find:

i. ∠DAB

ii. ∠BDC

	Statement	Reason
1.	∠BED = 65°	Given.
2.	∠BAD = ∠BED = 65°	Angles in the same segment are equal.
3.	∠ADB = 90°	Angle in a semicircle is right angle.
4.	∠CDA + ∠DAB = 180° ∠BDC + 90° + 65° = 180° ∠BDC = 25°	
5.	∠CDB = ∠DBA = 25°	Interior alternate angles.

(b) Given:

i. ∠SPR = x°

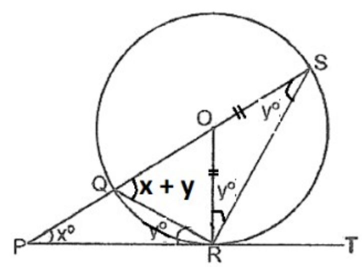
ii. ∠QRP = Y°

iii. PT is a tangent.

iv. SQ is a diameter.

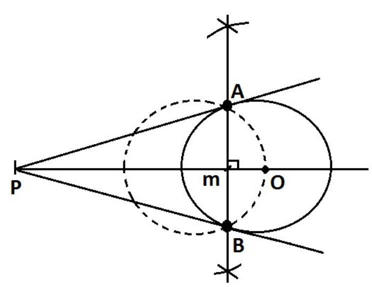
T.P.T : ∠ORS = Y°

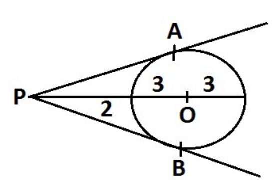
To find : Expression connecting x and y.



	Statement	Reason
1.	∠SPR = x°	Given.
2.	∠QRP = y°	Given.
3.	∠QSR = y°	By alternate segment theorem.
4.	OS = OR	Radii of the same circle.
5.	∠ORS = OSR = y° Hence proved.	Angles opposite equal sides are equal.
6.	∠SRQ = 90°	Angle in a semicircle is a right angle.
7.	∠SQR = ∠QPR + ∠QRP = x + y	By exterior angle theorem.
8.	In Δ SRQ, x + y + y + 90° = 180° x + 2y = 90°	Sum of the angles of a triangle is 180°.

(c)





PA = PB = 4cm

Answers of Practice Paper 12

Section I

Answer 1.

(a) $x^2 + 2x + \frac{1}{3} = 0$

$3x^2 + 6x + 1 = 0$

a = 3, b = 2, c = 1

$D = b^2 - 4ac = 2^2 - (4 \times 3 \times 1) = 4 - 12 = \underline{-8}$

∴ The roots are imaginary.

(b) $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$C = \begin{bmatrix} 2 & 3 \\ 1 & -11 \end{bmatrix}$

∴Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

BA = C

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -11 \end{bmatrix}$

$\begin{bmatrix} a+2b & 3b-a \\ c+2d & 3d-c \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -11 \end{bmatrix}$

Since the matrices are equal:

a + 2b = 2

$\underline{-a + 3b} = \underline{3}$

5b = 5

$\underline{b} = \underline{1}$

Substituting b = 1 in a + 2b = 2

a + 2b = 2

a + 2 = 2

∴ a = $\underline{0}$

c + 2d = 1

$\underline{-c + 3d} = \underline{-11}$

5d = -10

d = $\underline{-2}$

Substituting d = -2 in eq: c + 2d = 1

c + 2 (-2) = 1

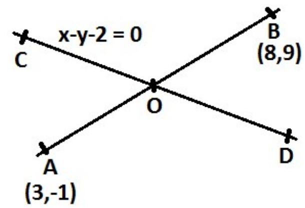
c – 4 = 1

$\underline{c} = \underline{5}$

$B = \begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}$

(c) A (x, y) = (3, -1) – (x_1, y_1)

B (x, y) = (8, 9) – (x_2, y_2)



Equation of line AB:

$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$

$\frac{y+1}{x-3} = \frac{9+1}{8-3}$

$\frac{y+1}{x-3} = \frac{10}{5}$

y + 1 = 2x – 6

2x – y = 7…………………… (i)

Equation of line CD is given as: x – y = 2 …………………… (ii)

Solving eq i and ii:

2x – y = 7........(i)

x – y = 2.......(ii)

x = 5

Substituting x = 5 in equation (i)

x – y = 2

5 – y = 2

Y = 5 – 2

Y = 3

Point of intersection = (5, 3) – ($\propto, \beta$)

Answer 2.

(a) $-1 \leq \frac{x}{2} - \frac{4}{3}$; $\frac{x}{2} - \frac{4}{3} < \frac{1}{6}$

$-1 \leq \frac{3x-8}{6}$; $\frac{x}{2} - \frac{4}{3} < \frac{1}{6}$

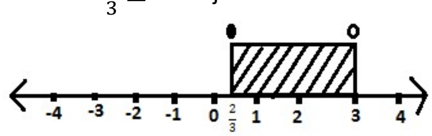
$-6 \leq 3x - 8$; $\frac{3x-8}{6} < \frac{1}{6}$

$-6 + 8 \leq 3x$; $3x < 1 + 8$

$+2 \leq 3x$; $3x < 9$

x < 3

set = {x : X ε R > $\frac{2}{3} \leq x < 3$}



(b) x = Rs 300 per month

n = 2 years = 24 months

M.V. = Rs 8100

r = ?

MV = n x + I;

8100	=	(24 X 300) + I;
<u>900</u>	=	<u>I</u>
I	=	$\frac{n\,x\,r\,(n+1)}{2400}$
900	=	$\frac{300\,X\,24\,X\,25\,X\,r}{2400}$
r	=	<u>12%</u>

(c) Given:

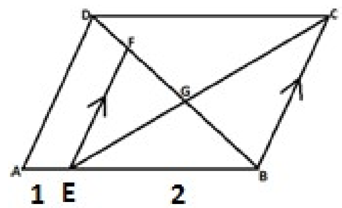
i. ABCD is a parallelogram.

ii. E is a point on AB.

iii. CE intersects the diagonal BD at G

iv. EF is parallel to BC.

v. AB : EB = 1 : 2,



<u>To find:</u>

i. EF : AD.

ii. Area of ΔBEF : Area of ΔABD.

Statement	Reason
1. In ΔBEF &ΔBAD,	
a. ∠BEF = ∠BAD	Corresponding∠.
b. ∠BFE = ∠BDA	Corresponding∠.
c. ΔBEF ~ΔBAD	By A.A test of similarity
2. $\therefore \frac{BE}{BA} = \frac{EF}{AD} = \frac{BF}{BD}$	Corresponding sides of similar Δs.
$\frac{BE}{BA} = \frac{EF}{AD}$	
$\frac{2}{2+1} = \frac{EF}{AD}$	
$\therefore \frac{EF}{AD} = \frac{2}{3}$	
EF : AD = 2 : 3	
3. $\frac{A\,(\Delta BEF)}{A\,(\Delta ABD)} = \frac{EF^2}{AD^2} = \frac{2^2}{3^2} = \frac{4}{9}$	Areas of two similar Δs are proportional to the squares on their corresponding sides.

Answer 3.

(a) $\therefore$ L.H.S = $\frac{\sin\theta\ \tan\theta}{1-\cos\theta} = \frac{\sin\theta\ \frac{\sin\theta}{\cos\theta}}{1-\cos\theta} = \frac{\sin^2\theta}{\cos\theta\ (1-\cos\theta)}$

$= \frac{1-\cos^2\theta}{\cos\theta\ (1-\cos\theta)} = \frac{(1+\cos\theta)\ (1-\cos\theta)}{\cos\theta(1-\cos\theta)} = \frac{1+\cos\theta}{\cos\theta}$

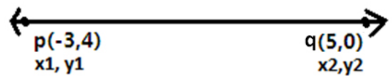
$= \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta}$ = sec θ + 1

R.H.S = sec θ + 1

<u>L.H.S = R.H.S</u>

<u>Hence proved</u>

(b)



$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{y - 4}{x + 3} = \frac{0 - 4}{5 + 3}$

$\frac{y - 4}{x + 3} = \frac{-4}{8}$

$2y - 8 = -x - 3$

$x + 2y - 8 + 3 = 0$

$x + 2y - 5 = 0$

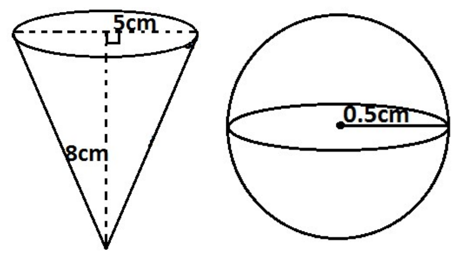
(c) $\frac{1}{4}^{th}$ volume of cone = n × volume of spheres

$\frac{1}{4} \times \frac{1}{3}\pi r^2 h = n \times \frac{4}{3}\pi r^3$

$\frac{1}{12} \times \frac{22}{7} \times 5 \times 5 \times 8 = n \times \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10}$

$n = \frac{5 \times 5 \times 8 \times 3 \times 10 \times 10 \times 10}{12 \times 4 \times 5 \times 5 \times 5}$

$= \underline{\underline{100}}$



Ans:- Number of lead shots are 100

Answer 4.

(a) Total no. of outcomes (n) = 52

i. No. of favourable outcomes (m) = 13

∴Pprobability of getting a diamond card,P (E) = $\frac{m}{n} = \frac{13}{52} = \frac{1}{4}$

ii. No. of favourable outcomes (m) = $3 \times 2 = \underline{6}$

Probability of getting a black card, P (E) = $\frac{m}{n} = \frac{6}{52} = \frac{3}{26}$

iii. No. of favourable outcomes (m) = $26 - 2 = \underline{24}$

Probability of getting red card nor queen, P (E) = $\frac{m}{n} = \frac{24}{52} = \frac{6}{13}$

(b) Factor = x – 1

Since (x – 1) is a factor,

0 = x – 1

x = 1

Since it is a factor,

$f(x) = x^3 - ax^2 - 13x + b$

$f(1) = 1^3 - a(1)^2 - 13(1) + b$

$0 = 1 - a - 13 + b$

$a - b = -12$……………………………………………(i)

factor = (x + 3)

since (x + 3) is a factor,

0 + x + 3,

x = –3

$f(x) = x^3 - ax^2 - 13x + b$

f (3)	=	$(-3)^3 - (-3)^2\, a^2 - (13 \times -3) + b$
0	=	$-27 - 9a + 39 + b$
0	=	$12 - 9a + b$
$9a - b$	=	12……………………………………………(ii)

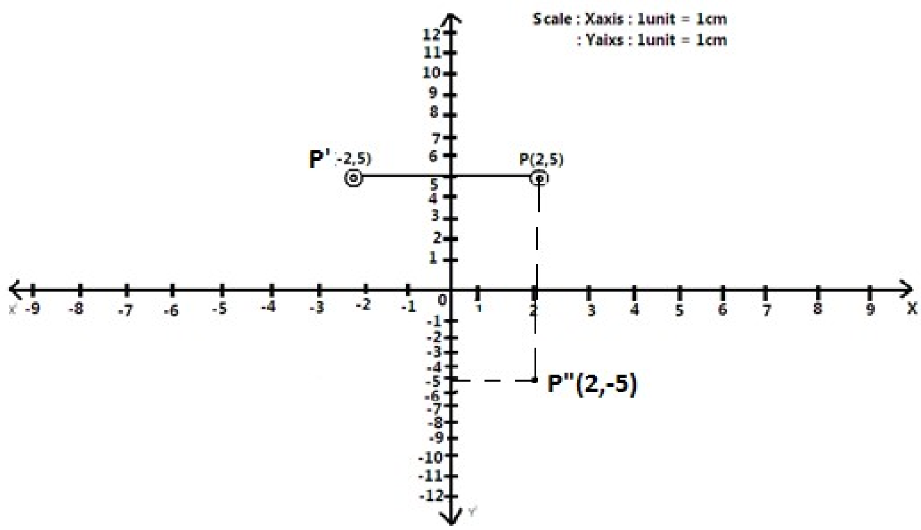
Equating (i) & (ii) simultaneously: -

$9a - b$	=	12
$a - b$	=	-12
$8a$	=	24
$\therefore a$	=	3

Substituting a = 3 in equation (i)

$a - b$	=	-12
$3 - b$	=	-12
$-b$	=	$-12 - 3$
$-b$	=	-15
b	=	15
a	=	<u>3</u>
b	=	<u>15</u>

(c)



i. P (a, b)	=	(2, 5)
ii. P^{II} (x, y)	=	(+2, -5)

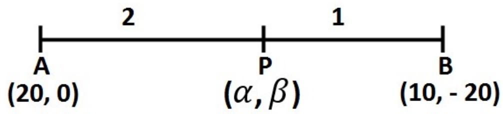
iii. Reflection in origin

iv. P	=	(2, 5) $= (x_1, y_1)$
p^{I}	=	(-2, 5) $= (x_2, y_2)$
$\frac{y-y_1}{x-x_1}$	=	$\frac{y_2 - y_1}{x_2 - x_1}$
$\frac{y-5}{x-2}$	=	$\frac{5-5}{-2-2}$
$\frac{y-5}{x-2}$	=	$\frac{0}{-4}$
$\frac{y-5}{x-2}$	=	0
$y - 5$	=	0

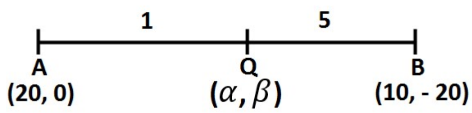
Section II

Answer 5.

(a)



$$\alpha = \frac{mx_2 + nx_1}{m+n} \quad ; \quad \beta = \frac{my_2 + ny_1}{m+n}$$
$$= \frac{2\,(10) + 1\,(20)}{2+1} \quad ; \quad = \frac{2\,(-20) + 1\,(0)}{2+1}$$
$$= \frac{20+20}{3} \quad ; \quad = \frac{-40+0}{3}$$
$$= \frac{40}{3} \quad ; \quad = \frac{-40}{3}$$
$$P = \left(\frac{40}{3}, \frac{-40}{3}\right)$$



$$\alpha = \frac{mx_2 + nx_1}{m+n} \quad ; \quad \beta = \frac{my_2 + ny_1}{m+n}$$
$$= \frac{1\,(10) + 5\,(20)}{1+5} \quad ; \quad = \frac{1\,(-20) + 5(0)}{1+5}$$
$$= \frac{10+100}{6} \quad ; \quad = \frac{-20}{6}$$
$$= \frac{110}{6} \quad ; \quad = \frac{-10}{3}$$
$$= \frac{55}{3}$$
$$Q = \left(\frac{55}{3}, \frac{-10}{3}\right)$$

(b) Cost of 2x articles = Rs. 5x + 54

Cost of 1 article = Rs. $\frac{5x+54}{2x}$

Cost of x+2 articles = 10x – 4

Cost of 1 articles = $\frac{10x-4}{x+2}$

Since articles are similar,

$$\frac{5x+54}{2x} = \frac{10x-4}{x+2}$$

(5x + 54) (x + 2) = 2x (10x – 4)

$5x^2 + 10x + 54x + 108 = 20x^2 - 8x$

$0 = 20x^2 - 5x^2 - 8x - 10x - 54x - 108$

$0 = 15x^2 - 72x - 108$

$0 = 5x^2 - 24x - 36$

a = 5

b = -24

$$c = -36$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(24) \pm \sqrt{(-24)^2 - 4(5)(-36)}}{2(5)}$$

$$= \frac{24 \pm \sqrt{576+720}}{10}$$

$$= \frac{24 \pm 36}{10}$$

$$x = \frac{24+36}{10} \quad ; \quad x = \frac{24-36}{10}$$

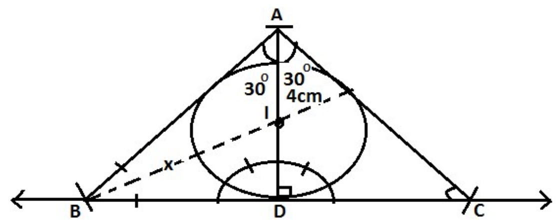
$$x = 6 \quad ; \quad x = -1.2$$

Ignoring the negative sign.

$x = \underline{6}$

(c) Radius of in circle = 1.3cm

(first draw base line, mark a point D anywhere in centre, draw perpendicular at D and cut A at 4 cm. From A make 30^{o} on both sides to get B and C. Then draw incircle.)



Answer 6.

(a)

$$2x - 4y = 5$$

$$4y = 2x - 5$$

$$y = \frac{x}{2} - \frac{5}{4}$$

$$\therefore m = \frac{1}{2}$$

$$mm^{l} = -1$$

$$\frac{1}{2} m^{l} = -1$$

$$m^{l} = -\underline{2}$$

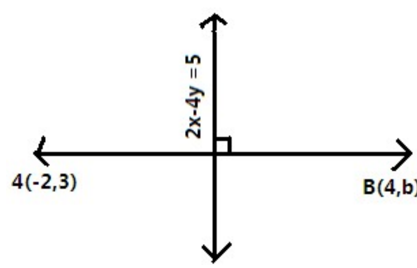
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-2 = \frac{b-3}{4+2}$$

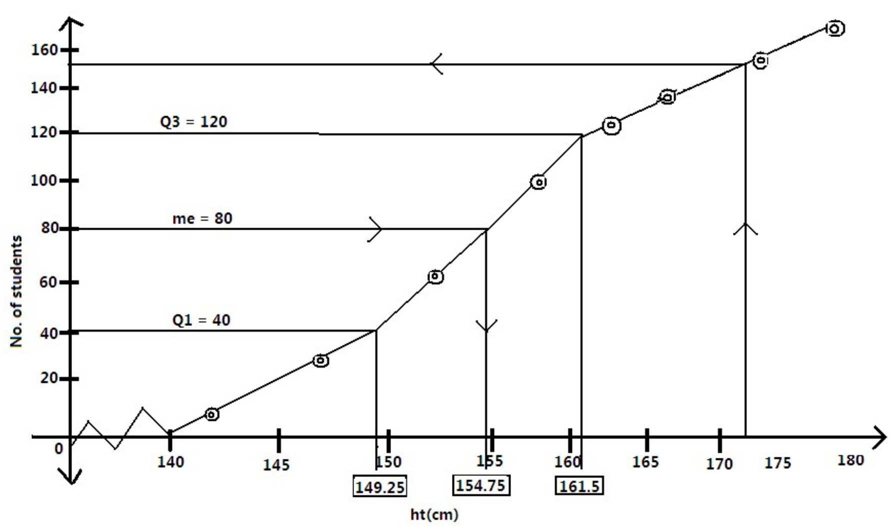
$$-12 = b - 3$$

$$b = -12 + 3$$

$$b = -9$$



(b)



C.I	f	c.f
140 – 145	12	12
145 – 150	20	32
150 – 155	30	62
155 – 160	38	100
160 – 165	24	124
165 – 170	16	140
170 – 175	12	152
175 – 180	8	160
	160	

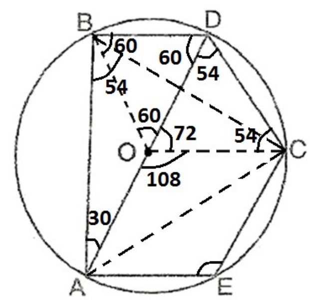
i. Me = x value of cf/2 = x value of 80 = 154.15

ii. Inter quarterly = $Q_3 - Q_1$ = 161.5 – 149.25 = 10.75

iii. Number of person whole height is above 172cm = 170 – 151= 19

Answer 7.

(a) Given:



i. BD is a side of a regular hexagon.

ii. DC is a side of a regular pentagon.

iii.AD is a diameter.

To find :

i. ∠ADC

ii. ∠BDA

iii. ∠ABC

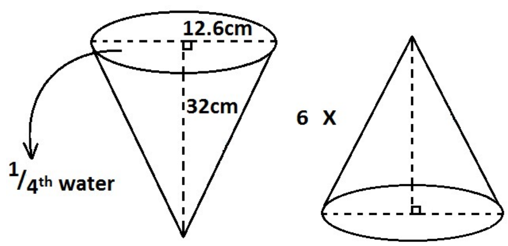
iv. ∠AEC.

Cons. : Join BO, CO, BC, AC.

	Statement	Reason
1.	BD is the side of a regular hexagon.	Given.
2.	Each interior angle of a regular hexagon $= \frac{(2n-4)\times 90°}{n} = \frac{(2\times 6-4)\times 90°}{6} = \frac{720°}{6} = 120°$	By formula.
3.	∠OBD $=$ ∠ODB $= \frac{120°}{2} = 60°$	OB and OD are angle bisector of each interior angle.
4.	∠BDA = ∠ODB = 60°	AOD is a straight line.
5.	∠BOD $= 180° - (60° + 60°) = 180° - 120° = 60°$	Sum of the angles of a triangle BOD is 180°.
6.	DC is the side of the regular pentagon	Given.
7.	Each interior angle of a regular pentagon $= \frac{(2n-4)\times 90°}{n} = \frac{(2\times 5-4)\times 90°}{n} = \frac{540°}{5} = 108°$	By formula.

8.	∠ODC	= ∠OCD $= \frac{108°}{2} = 54°$	OD and OC are angle bisectors of each interior angle.
9.	∠ADC	= ∠ODC = 54°	AOD is a straight line.
10.	∠DOC	= 180° – (54° + 54°) = 180° – 108° = 72°	Sum of the angles of a triangle DOC is 180°.
11.	∠AOC	= 180° – ∠DOC = 180° – 72° = 108°	Linear pair.
12.	∴ ∠ABC	$= \frac{1}{2}$∠AOC $= \frac{1}{2} \times 108°$ = 54°	Angles subtended at the centre is double that of the circumference.
13.	∠ABC	= ∠ADC = 54°	Angle in the same segment are equal.
14.	∠AEC	= 180° – ∠ABC = 180° – 54° = 126°	Opposite angles of a cyclic quadrilateral ABCE are supplementary.

(b)



$\frac{1}{4}^{th}$ volume of big cone = 6 × volume of small cone

$\frac{1}{4} \times \frac{1}{3}\pi r^2 h$ = 6 × volume of small cone

Volume of small cone s $= \frac{1}{6} \times \frac{1}{4} \times \frac{1}{3} \times \frac{22}{7} \times \frac{126}{10} \times \frac{126}{10} \times 32$ = $221.76\ cm^3$

(c) Given:

i. Segment MN is the diameter of the circle of radius 'r' with center O.

ii. Segment PM is a tangent to the circle at M.

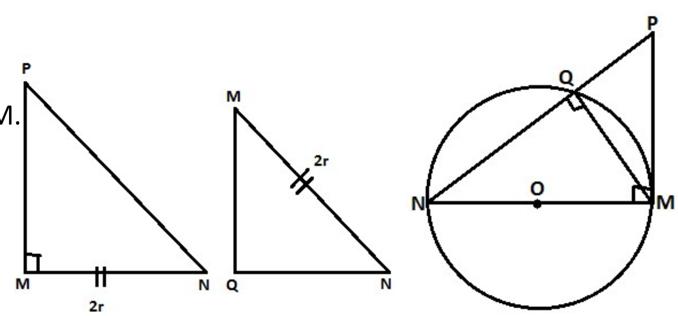
iii. Segment PN intersects the circle at Q.

PM = 7cm

PN = 14cm,

To Prove That :

PN.QN = $4r^2$. Find QN.



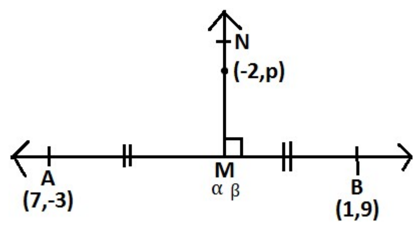
Statement	Reason
1. in ΔPMN & ΔMQN ∠N = ∠N ∠M = ∠Q	 Common∠. 90^0 given

ΔPMN ~ ΔMQN $\frac{PM}{MQ} = \frac{MN}{QN} = \frac{PN}{MN}$ $\frac{MN}{QN} = \frac{PN}{MN}$ MN X MN = PN X QN 2r X 2r = PN X QN 4r = PN X QN 2. pm^2 = PQ X PN 7 X 7 = PQ X 14 PQ = $\frac{7}{2}$ = 3.5 cm QN = PN – PQ = 14 – 3.5 = 10.5 cm	Corresponding sides of similar Δ. Proved Length of square of tangent is equal to the length of chord touching circumstances.

Answer 8.

(a) A = (7, 3) – (X_1, Y_1)

B = (1, 9) – (X_2, Y_2)



i. Slope of AB:

$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9+3}{7-1}$

$= \frac{12}{-6} = \underline{-2}$

For midpoint of AB, (M),

$\propto = \frac{x_2 - x_1}{2}$; $\beta = \frac{y_2 - y_1}{2}$

$\propto = \frac{7+1}{2}$; $\beta = \frac{9-3}{2}$

$\propto = \frac{8}{2}$; $\beta = \frac{6}{2}$

$\propto = \underline{4}$; $\beta = \underline{3}$

M ($\propto$, β) = (4 , 3)

ii. Equation of AB Line MN:

$mm^I = -1$

$-2m^I = -1$

$\therefore m^I = \frac{-1}{2}$

M = (4, 3) = (x_1, y_1)

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{2}(x - 4)$

$2y - 6 = x - 4$

$X - 2y = -6 + 4$

$\underline{X - 2y} = \underline{-2}$

iii. (–2, p) lies on the same line, hence substituting it in the eq we get:

$X - 2y = -2$

$-2-2p = -2$

$p = \underline{0}$

(b) Scale factor (k) = 1:200

i. Length of model = k X actual length

5 = $\frac{1}{200}$ X l

L = 5 X 200 = <u>1000m</u>

ii. area of model = k^2 X actual area

a^2 = $\frac{1}{200}$ X $\frac{1}{200}$ X 2,00,000 = $5m^2$

iii. volume of model = k^3X volume of ship

300 = $\frac{1}{200}$ X $\frac{1}{200}$ X $\frac{1}{200}$ X v

V = 300 X 200 X 200 X 200 = 2400000000l = <u>24, 00,000 m^3</u>

(c) Given:

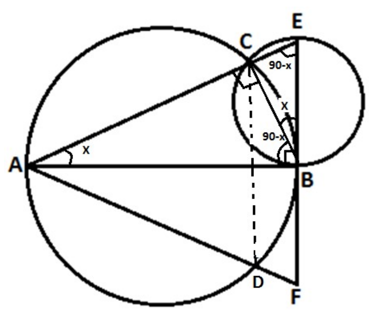
i. BE is the diameter of the circle, passing through B, C and E.

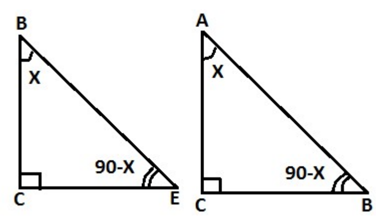
ii. AB is a tangent to the circle passing through B, C and E.

To prove that:

i. BE is a tangent to the circle passing through A, C and D.

ii. ΔBCE ~ ΔACB.





Statement	Reason
1. Angle ABE = 90	Given diameter BE is perpendicular to the given tangent of the circle.
2. ∴line EBF is a tangent to circle passing through ACD.	Diameter AB is perpendicular to the line EBF, from statement 1 above. Hence <u>Proved.</u>
3. ΔBCE & ΔACB Let ∠CAB = x ∠ ACB = 90 ∠ ABC = 90 – x ∠ EBA = 90^0 ∠ CBE = 90 – (90 – x) = x ∠CEB = 90 – x	 ∠ in a semicircle Sum of the angles of aΔ. Tangent given Addition property. Sum of the angles of a Δ is 180.

$\angle C = \angle C$; 90^0 $\angle CEB = \angle CBA$ $\Delta BCE \sim \Delta ACB$	 Both 90 – x from above. BY AAA test of similarity. Proved.

Answer 9.

(a) $x^2 + 2(m - 1)x + (m + 5) = 0$

$$\begin{aligned} a &= +1 \\ b &= 2(m-1) \\ c &= (m+5) \\ d &= b^2 - 4ac \\ &= (2m-2)^2 - 4(+1)(m+5) \quad = 4m^2 - 8m + 4 - 4m - 20 \\ &= 4m^2 - 12m - 6 \quad = m^2 - 3m - 4 \\ &= m^2 - 4m + m - 4 \quad = m(m-4) + 1(m-4) \\ &= (m-4)(m+1) \end{aligned}$$

$m + 1 = 0$ or $m - 4 = 0$

$m = -1$ or $m = 4$

$m = \underline{\underline{\{-1, 4\}}}$

(b) By component & dividend,

$$\frac{b+1}{b-1} = \frac{a+\sqrt{a^2-2ax}+a-\sqrt{a^2-2ax}}{a+\sqrt{a^2-2ax}-a+\sqrt{a^2-2ax}}$$

$$\frac{b+1}{b-1} = \frac{2a}{2(\sqrt{a^2-2ax})}$$

$$\frac{b+1}{b-1} = \frac{a}{(\sqrt{a^2-2ax})}$$

Squaring both sides,

$$\frac{b^2+2b+1}{b^2-2b+1} = \frac{a^2}{a^2-2ax}$$

By component & dividend

$$\frac{b^2+2b+1+b^2-2b+1}{b^2+2b+1-b^2+2b-1} = \frac{a^2+a^2-2a}{a^2-a^2+2ax}$$

$$\frac{2(b^2+1)}{2(2b)} = \frac{2a(a-x)}{2ax}$$

$$\frac{b^2+1}{(2b)} = \frac{(a-x)}{x}$$

$$x(b^2+1) = 2b(a-x)$$

$$x(b^2+1) = 2ab - 2bx$$

$$x(b^2+1) + 2bx = 2ab$$

$$x[b^2 + 1 + 2b] = 2ab$$

$$x = \frac{2ab}{b2 + 2b + 1}$$

(c) $S_n = 5n^2 - 8n$

$S_1 = 5(1)^2 - 8 \times 1 = 5 - 8 = -3$

$S_2 = 5(2)^2 - 8 \times 2 = 20 - 16 = 4$

$\therefore T_2 = S_2 - S_1 = 4 - (-3) = 4 + 3 = 7$

And $a = S_1 = -3$

$d = T_2 - T_1$

$= 7 - (-3) = 7 + 3 = 10$

$\therefore$ A.P. $= -3, 7, 17, \ldots\ldots\ldots\ldots\ldots$

And $T_{15} = a + 14d = -3 + 14 \times 10 = -3 + 140 = \underline{\underline{137}}$

Answer 10.

(a) $f(x) = x^3 - px^2 + x + 6$

If $x - 3 = 0$

$x = 3$

Using remainder Theorem,

$\therefore f(x) = x^3 - px^2 + x + 6 = (3)^3 - p(3)^2 + 3 + 6$

$= 27 - 9p + 9 = 36 - 9p$ ---- (1)

$g(x) = 2x^3 - x^2 - (p + 3)x - 6 = 2(3)^3 - (3)^2 - (p + 3)3 - 6$

$= 2(27) - 9 - 3p + 9 - 6 = 54 - 9 - 3p - 9 - 6$

$= 30 - 3p$ ------ (2)

$\therefore$ Since the remainder's are equal,

(1) = (2)

$\therefore 36 - 9p = 30 - 3p$

$36 - 30 = -3p + 9p$

$6 = 6p$

$p = \frac{6}{6} = \underline{\underline{1}}$

(b) A boy spends Rs.10 on first day,

Rs.20 on second day

Rs.40 on third day and so on

G.P. is 10 + 20 + 40 + …… 12 terms

Here a = 10, r = 2 and n = 12 terms (r > 1)

$S_{12} = \frac{a(r^n - 1)}{r - 1} = \frac{10(2^n - 1)}{2 - 1} = \frac{10(2^{12} - 1)}{1} = \underline{\underline{10(2^{12} - 1)}}$

(c) $\therefore \frac{(a - 2b - 3c + 4d) + (a + 2b - 3c - 4d)}{(a - 2b - 3c + 4d) - (a + 2b - 3c - 4d)} = \frac{(a - 2b + 3c - 4d) + (a + 2b + 3c + 4d)}{(a - 2b + 3c - 4d) - (a + 2b + 3c + 4d)}$

$\therefore \frac{a - 2b - 3c + 4d + a + 2b - 3c - 4d}{a - 2b - 3c + 4d - a - 2b + 3c + 4d} = \frac{a - 2b + 3c - 4d + a + 2b + 3c + 4d}{a - 2b + 3c - 4d - a - 2b - 3c - 4d}$

$\therefore \frac{2a - 6c}{-4b + 8d} = \frac{2a + 6c}{-4b - 8d}$

$\therefore \frac{2(a - 3c)}{4(b - 2d)} = \frac{2(a + 3c)}{4(b + 2d)}$

$$\frac{a-3c}{b-2d} = \frac{a+3c}{b+2d}$$

$$\frac{a-3c}{a+3c} = \frac{b-2d}{b+2d} \quad \text{(By alternendo)}$$

$$\frac{(a-3c)+(a+3c)}{(a-3c)+(a+3c)} = \frac{(b-2d)+(b+2d)}{(b-2d)-(b+2d)}$$

$$\frac{a-3c+a+3c}{a-3c-a-3c} = \frac{b-2d+b+2d}{b-2d-b-2d}$$

$$\frac{2a}{-6c} = \frac{2b}{-4d}$$

$$\frac{a}{-3c} = \frac{b}{-2d}$$

$$\frac{a}{-3c} = \frac{b}{-2d}$$

$$\frac{a}{3c} = \frac{b}{2d}$$

$$2ad = 3bc$$

Hence proved.

Answer 11.

(b) **LHS**

$= \frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{1-\mathrm{co}} + \frac{\sin\theta}{1+\mathrm{co}}$

$= \frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\ )}{1-\sin^2\theta} + \frac{\sin\theta(1+\cos\theta)+\sin\theta(1-\sin\ )}{1-\cos^2\theta}$

$= \frac{\cos\theta[(1+\sin\theta)+(1-\sin\theta)]}{\cos^2\theta} + \frac{\sin\theta[(1+\cos\theta)+(1-\cos\theta)]}{\sin^2\theta}$

$= \frac{1+\sin\theta+1-\sin\theta}{\mathrm{co}} + \frac{1+\cos\theta+1-\cos\theta}{\sin\theta}$

$= \frac{2}{\cos\theta} + \frac{2}{\sin\theta}$

= 2cosecΘ + 2secΘ

= 2(cosecΘ + secΘ)

LHS = RHS,

proved.

(b) Mean = 7

Mean $= \frac{sum\ of\ total\ numbers}{Total\ no.}$

$7 = \frac{6+4+5.5+8+9.5+b}{7}$

49 = 41 + b

b = 8

If each gets reduced by 1, there will be deduction of 7 in total sum:-

New mean $= \frac{sum\ of\ total\ numbers}{Total\ no.}$

$= \frac{49-7}{7} = \frac{42}{7} = 6$

(c) In ΔABC,

$$\tan 30° = \frac{AC}{BC}$$

$$\tan 30° = \frac{h-5}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{h-50}{y}$$

$$y = \sqrt{3}\,(h-50) \text{ .... (i)}$$

In ΔBFC,

$$\tan 60° = \frac{CF}{BC}$$

$$\tan 60° = \frac{50+h}{y}$$

$$\sqrt{3} = \frac{50+h}{y}$$

$$y = \frac{50+h}{\sqrt{3}} \text{ .... (ii)}$$

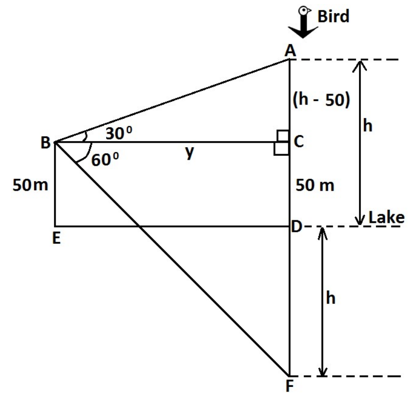
Equating (i) and (ii):-

$$\sqrt{3}\,(h-50) = \frac{50+h}{\sqrt{3}}$$

$$2h - 150 = 50 + h$$

$$2h = 200$$

$$\underline{\underline{h}} = \underline{\underline{100m}}$$



Answers of Practice Paper 13

Section I

Answer 1.

(a) $2x - 3 = \sqrt{(2x^2 - 2x + 21)}$

$(2x-2)^2$	=	$2x^2 - 2x + 21$
$4x^2 - 12x + 9$	=	$2x^2 - 2x + 21$
$4x^2 - 2x^2 - 12x + 2x + 9 - 21$	=	0
0	=	$2x^2 - 10x - 12$
0	=	$x^2 - 5x - 6$
0	=	$x^2 - 6x + x - 6$
0	=	$x(x - 6) + 1(x - 6)$
0	=	$(x + 1)(x - 6)$
X	=	<u>{(-1, 6)}</u>

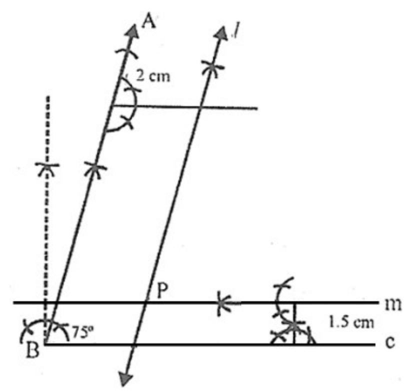
(b) Given: ABC = 75°, distance of 2 cm from AB and 1.5 cm from BC.

To find: Point P

Construction:

i. Draw a ray BC.
ii. At B, draw a ray BA making an angle of 75° with BC.
iii. Draw a line l parallel to AB at a distance of 2 cm.
iv. Draw another line m parallel to BC at a distance of 1.5 cm. which intersects m at P.

∴ P is the required point.



(c)

Case – I : Car	Case – II : Train
D = 216 km S = x km/h T = $\left(\frac{216}{x}\right)$hrs	D = 208 km S = (x + 16) km/h T = $\left(\frac{208}{x+16}\right)$hrs

The train takes 2hrs less than the car

$$\therefore \frac{216}{x} - \frac{208}{x+16} = 2$$

$$\frac{216(x+16) - 208x}{x(x+16)} = 2$$

$$216x + 3456 - 208x = 2x^2 + 32x$$

$$3456 + 8x = 2x^2 + 32x$$

$$0 = 2x^2 + 24x - 3456$$

$$0 = x^2 + 12x - 1728$$

$$0 = x^2 + 48x - 36x - 1728$$

$$0 = x(x + 48) - 36(x + 48)$$

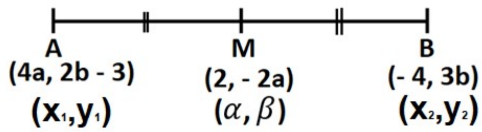
$$0 = (x - 36)(x + 48)$$

$$\therefore 0 = x - 36 \text{ or } 0 = x + 48$$

$\therefore$ x = 36   or x   = –48
x = 36km/h or x   = –48km/h
Ignoring the negative value:
Speed of car   = $\underline{\underline{36kmh^{-1}}}$
Speed of train = x + 16 = 36 + 16 = $\underline{\underline{52\ kmh^{-1}}}$

Answer 2.

(a)



α	$= \frac{x_1 + x_2}{2}$	;	β	$= \frac{y_1 + y_2}{2}$
2 × 2	= 4a + (– 4)	;	– 2a × 2	= 2b – 3 + 3b
4 + 4	= 4a	;	– 2 (2) × 2	= 2b + 3b – 3
$\frac{8}{4}$	= a	;	– 8 + 3 b	= 5b
$\underline{\underline{2}}$	= a	;	b	= $\underline{\underline{-1}}$

(b)
x = Rs. 2000 per month
r = 10%
M.V = Rs. 67750
n =?

$M.V = nx + \frac{nxr\ (n+1)}{2400}$

$67750 = 2000n + \frac{2000\ X\ n\ X\ 10\ (n+1)}{2400}$

$67750 = 2000n + \frac{50(n+1)n}{6}$

$406500 = 12000n + 50n^2 + 50n$

$0 = 50n^2 + 50n + 12000n – 40, 6,500$

$0 = n^2\ 241n – 8130$

$0 = n^2 + 271n – 30n – 8130$

0 = n (n + 171) – 30 (n + 271)

0 = (n + 271) (n – 30)

$\underline{n}$ = $\underline{30\ months}$

(c) Total no of outcome (n) = {(1,1) (1,2) (1 ,3) (1, 4) (1,5) (1,6)
(2, 1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3, 1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4, 1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

n = 36

i. Total of 9:

No. of favorable outcome (m) = {(3,6) (4,5) (5,4) (6,3) = 4

Probability of getting a sum of 9:

P(E) = $\frac{m}{n}$ = $\frac{4}{36}$ = $\frac{1}{9}$

ii. A doublet:

No of favorable outcome (m) = (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6) = 6

Probability of getting a doublet:

P (E) = $\frac{m}{n}$ = $\frac{6}{36}$ = $\frac{1}{6}$

iii. 5 on one dice and 6 on the other dice:

No of favorable outcome (m) = (5,6) (6,5) = 2

Probability of getting 5 on one dice & 6 on other:

P (E) = $\frac{m}{n}$ = $\frac{2}{36}$ = $\frac{1}{18}$

iv. Product of 2 number is 12:

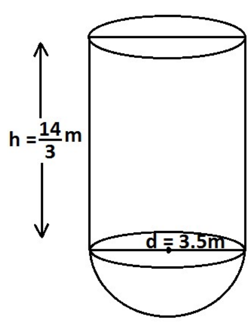
No of favorable outcome (m) = {(2,6) (3,4) (4,3) (6,2)} = 4

Probability of getting product of 2 nos is 12:

P (E) = $\frac{m}{n}$ = $\mathbf{\frac{4}{36}}$ = $\mathbf{\frac{1}{9}}$

Answer 3.

(a)



Volume of cylinder = $\pi r^2 h$ = $\frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} \times \frac{14}{3}$ = $\frac{539}{12}$ m^3

Volume of hemisphere = $\frac{2}{3}\pi r^3$ = $\frac{2}{3} \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20} \times \frac{35}{20}$ = $\frac{539}{48}$ m^3

Total Capacity = $\frac{539}{12} + \frac{539}{48}$ = $\frac{2695}{48}$ = $\underline{\underline{56.15\ m^3}}$

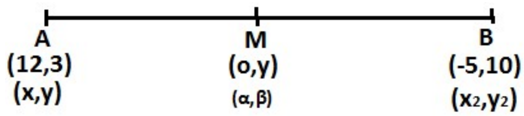
Internal surface area of hemisphere = $2\pi r^2$

= $2 \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20}$ = $\frac{77}{4}$

Internal surface area of cylinder $= 2\pi rh = 2 \times \frac{22}{7} \times \frac{35}{20} \times \frac{14}{3} = \frac{154}{3}$

Total T. S. A $= \frac{77}{4} + \frac{154}{3} = \frac{847}{12} = \underline{\underline{70.58\ m^2}}$

(b)



A (x_1, y_1) = (12, 3)

B (x_2, y_2) = (-5, 10)

M (α, β) = (o, y)

β $= \frac{y_1 + y_2}{2} = \frac{3+10}{2} = \frac{13}{2} = 6.5$

M (α, β) $= (0, 6\frac{1}{2})$

(c) If x – 2 is a factor,

X – 2 = 0

X = 2

Remainder = 2

f (x) $= 2x^3 - x^2 - px - 2$

f (2) $= 2(2)^3 - (2)^2 - p(2) - 2$

since (x – 2) is a factor,

f (x) = 0

0 = (2 X 8) – 4 – 2p – 2

0 = 16 – 6 – 2p

p = 5

f (x) $= 2x^3 - x^2 - 5x - 2$

$2x^2 + 3x + 1$

$x-2\overline{)\,2x^3 - x^2 - 5x - 2}$

$-2x^3 - 4x^2$

\- +

$3x^2 - 5x$

$-3x^2 - 6x$

x - 2

-x – 2

0

f(x) $= (x-2)(2x^2 + 3x + 1)$

f(x) $= (x-2)(2x^2 + 2x + x + 1)$

f(x) $= (x-2)[2x(x+1) + 1(x+1)]$

f(x) $= (x-2)[(2x+1)(x+1)]$

f(x) $= \underline{(x-2)(x+1)(2x+1)}$

Answer 4.

(a) $\frac{\sin A}{1+\cos A}$ = cosec A – cot A

R.H.S:

= Cosec A – cot A $\quad = \frac{1}{\sin A} - \frac{\cos}{\sin A}$

$= \frac{1-\cos A}{\sin A} \times \frac{1+\cos A}{1+\cos A} \quad = \frac{1-cos^2 A}{\sin A\,(1+\cos A)}$

$= \frac{sin^2 A}{\sin A\,(1+\cos A\,)} \quad = \frac{\sin A}{1+\cos A}$

L.H.S = R.H.S

Hence proved

(b) $2x - 3 < x + 2 < 3x + 5,\ x \epsilon R$

$2x - 3 < x + 2 \quad ; \quad x + 2 < 3x + 5$

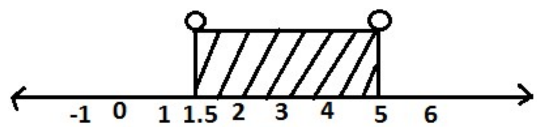
$-3 - 2 < x - 2x \quad ; \quad x - 3x < 5 - 2$

$-5 < -x \quad ; \quad -2x < 3$

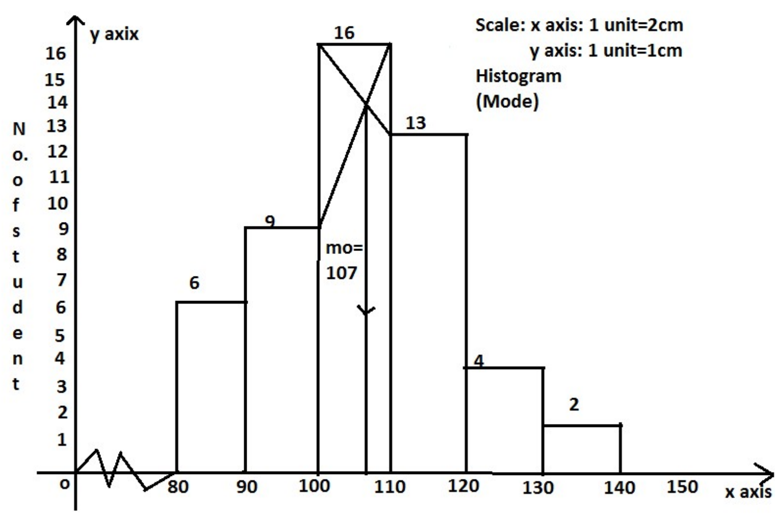
$5 > x \quad ; \quad 2x > 3$

$5 > x \quad ; \quad x > 1.5$

$5 > x > 1.5$ or $1.5 < x < 5$



(c)



1 Q, score	**80-90**	**90-100**	**100-110**	**110-220**	**120-130**	**130-140**	**140-150**
No. of student	6	9	16	13	3	2	50

Mode = 107

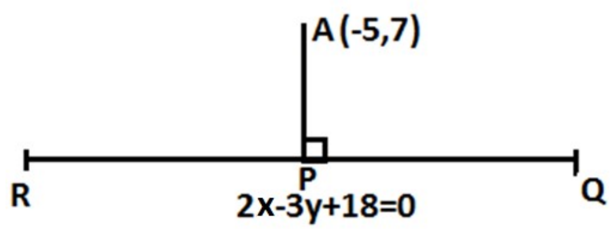
Section II

Answer 5.

(a)

Investment 1		Investment 2	
Fv	= 100	Fv	= 10
n	=?	n	=?
r	= 8%	r	= 11%
mv	= 95	mv	=?
D	=?	I	=?
Sp	= 105	D	= old + 42
			= 448 + 42 = 490
Mv X n	= I	I	= 5880
n	$= \frac{5320}{95}$	D	$= \frac{Fvnr}{100}$
	= 56	490	$= \frac{10 \times n \times 11}{100}$
D	$= \frac{Fvnr}{100}$	n	= 445.45
	$= \frac{100 \times 56 \times 8}{100}$		= 445
	= 448	mv × n	= I
Sale proceeds	= 56 × 105	mv × 445	= 5880
	= 5880	mv	= 13.20

(b) Line RQ:



2x – 3y + 18 = 0 ……………………….(i)

3y = 2x + 18

y $= \frac{2x}{3} + \frac{18}{3}$

m $= \frac{2}{3}$

Slope of AP:

Mm^1 = -1

$\frac{2}{3} m^1$ = -1

$m^1 = \frac{-3}{2}$

equation of AP:

A = (-5, 7) – (x_1, y_1)

m $= \frac{-3}{2}$

$y - y_1$ = $m(x - x_1)$

y – 7 $= \frac{-3}{2}$ (x + 5)

2y – 14 = -3x -15

3x + 2y – 14 + 15 = 0

3x + 2y + 1 = 0 ……………………….(ii)

Coordinates of P: solve (i) and (ii)

2x – 3y = -18 ……………………….(i)

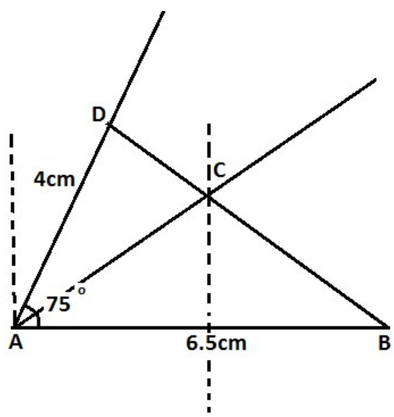
+ 3x + 2y = -1 ……………………….(ii)

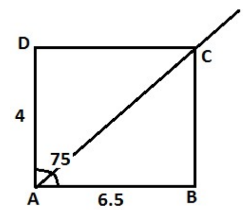
$5x - y = -19$ ....(iii)
$2x - 3y = -18$
$3x + 2y = -1$
$-x - 5y = -17$ ....(iv)

Solve (iii) and (iv)

$5x - y = -19$
$-5x - 25y = -85$
$-26y = -104$
$y = 4$
$-x - 5(4) = -17$
$-x = -17 + 20$
$x = -3$

(b)





Answer 6.

(a) According to the condition given:

$\frac{a}{6} = \frac{6}{b}$
$ab = 36$ ....(i)
$\frac{a}{b} = \frac{b}{48}$
$48a = b^2$
$48a = \left(\frac{36}{a}\right)^2$
$a^3 = \frac{36 \times 36}{48}$
$a^3 = 3 \times 3 \times 3$
$a = 3$
$b = \frac{36}{a} = \frac{36}{3} = 12$

(b) $f(x) = 3x^2 + 7x$

Let the number to be added be 'a'

$\therefore f(x) = 3x^2 + 7x + a$
$x - 3 = 0$

x = 3

∴ f(3) = $3(3)^2 + 7(3) + a$

When f(x) is divided by (x – 3 = 0), the remainder is 2

∴ 2 = 3(9) + 21+ a

2 = 48 + a

a = -46

∴ Number to be added = - 46

(c) Given:

i. In ΔABC, D and E are points on AB and AC such that DE ∥ BC.

ii. BC: DE = 5 : 4,

To find: The ratio of areas of trapezium BCED and ΔADE.

Statement	Reason
1. $\frac{DE}{BC} = \frac{4}{5}$	Given
2. In Δ ADE and ΔABC,	
a. ∠ADE = ∠ABC	Corresponding angle
b. ∠AED = $\angle ACB$	Corresponding angle
C. ΔADE ~ ΔACB.	By A.A test of similarity
3. $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} = \frac{4}{5}$	Corresponding sides of congruent triangles
4. $\frac{A(\Delta BCED)}{A(\Delta ADE)} = \frac{A\Delta ABC - A\Delta ADE}{A\Delta ADE}$	The area of two similar triangles is proportional to the squares on their corresponding sides.
$= \frac{BC^2 - DE^2}{DE^2}$ $= \frac{5^2 - 4^2}{4^2}$	
$= \frac{25-16}{16}$ $= \frac{9}{16}$	

Answer 7.

(a) x = a sin A cos C ; $\frac{n}{\sin A \cos C}$

y = a sin A sin C

z = a cos A

$x^2 + y^2 + z^2 = (a \sin A \cos C)^2 + (a \sin A \sin C)^2 + (a \cos A)^2$

$= a^2 \sin^2 A \cos^2 C + a^2 \sin^2 A \sin^2 C + a^2 \cos^2 A$

$= a^2 (\sin^2 A \cos^2 C + \sin^2 A \sin^2 C + \cos^2 A)$

$= a^2 [\sin^2 A (\cos^2 C + \sin^2 C) + \cos^2 A]$

$= a^2 [\sin^2 A (1) + \cos^2 A]$

$= a^2 (1)$

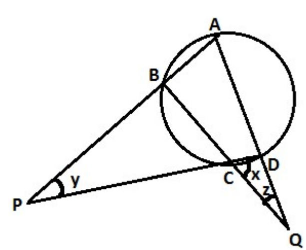
$= a^2$ proved

(b) Given:

i. ABCD is a cyclic quadrilateral.

ii. $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$,

To find : values of x, y and z.



Statement	Reason
1) $\frac{y}{4} = \frac{z}{5}$ $y = \frac{4z}{5}$ ----- (i) $\frac{x}{3} = \frac{y}{4}$ $\frac{x}{3} = \frac{4z}{5} X \frac{1}{4}$ $\frac{x}{3} = \frac{z}{5}$ $x = \frac{3z}{5}$ ----- (ii)	Given Given
2) ∟BAD = x	Ent. ∟ of a cyclic quadrilateral is equal to int. opposite angle.
3) ∟CDA = x + z	Ent. Angle of Δ CQD
4) In Δ ADP, ∟P = y ∟A = x ∟D = x + z $X + y + x + z = 180^o$ ____(iii) Substituting (i) and (ii) in eq (iii) x + y + z = 180 $\frac{3z}{5} + \frac{4z}{5} + z = 180$ $\frac{3z+4z+5z}{5} = 180$ 12z = 180X 5 $z = 75^o$ $y = \frac{4z}{5} = 60^o$ $x = \frac{3 X x}{5} = 45^o$	 Given From st 2 above From st 3 above Sum of the angles of a Δ

(c) 2y = 8 + x ; $y = 4 + \frac{x}{2}$;

	A	B	C
X	-1	0	1
Y	3 ½	4	4 ½
(x,y)	(-1, 3 ½)	(0, 4)	(1, 4 ½)

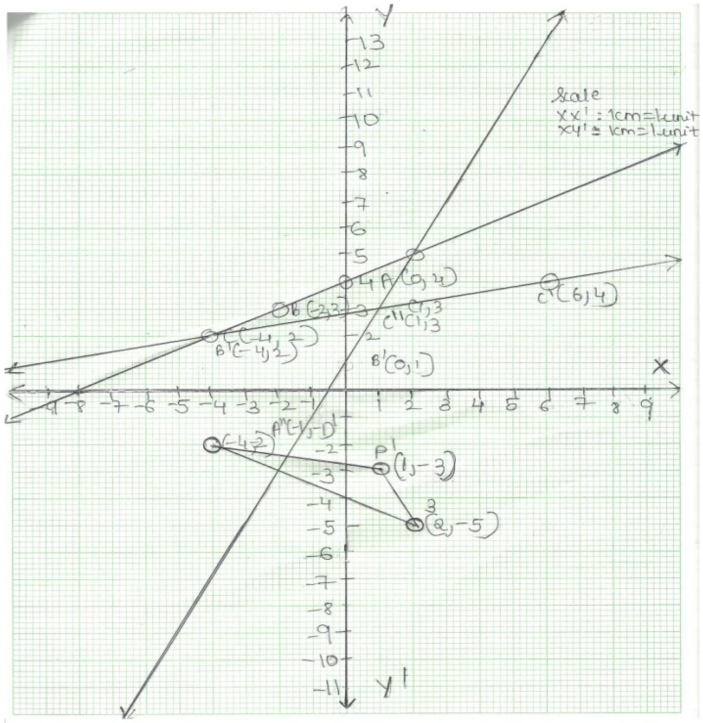
5y = 14 + x ; $y = \frac{14}{5} + \frac{x}{5}$;

	D	E	F
X	-1	0	1
Y	$\frac{13}{5}$	$\frac{14}{5}$	3
(x,y)	$(-1, 2\frac{3}{5})$	$(0, 2\frac{4}{5})$	(1, 3)

y = 1 + 2x ; y = 1 + 2x

	G	H	I
X	-1	0	1
Y	-1	1	3
(x,y)	(-1, -1)	(0, 1)	(1, 3)

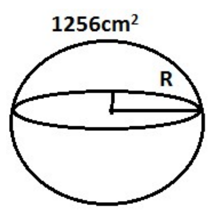
Graph for 7(a) and (b) – m_x. A (2,5) – A′ (2, -5) ; m_x. B (1, 3) – B′ (1 , -3) ; m_x. C (-4, 2) – C′ (-4, -2)

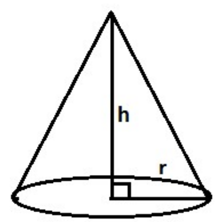


Answer 8.

(a)

TSA of sphere	= 1256
$4\pi R^2$	= 1256
4 X 3.14 X R^2	= 1256
R^2	= 100
R	= 10 cm
Volume of sphere	= n X volume of cone
$\frac{4}{3}\pi R^3$	= n x $\frac{1}{3}\pi R^2$ h
$4R^3$	= n X r^2 h
4 X 10^3	= n X 2.5^2 X 8
n	= <u>80</u>





(b) $\frac{\sqrt{36x+1}+6\sqrt{x}}{\sqrt{36x+1}-6\sqrt{x}} = \frac{9}{1}$; By componendo – dividendo

$\frac{\sqrt{36x+}+6\sqrt{x}+\sqrt{36x+1}-6\sqrt{x}}{\sqrt{36x+1}+6\sqrt{x}-\sqrt{36x+1}+6\sqrt{x}} = \frac{9+1}{9-1}$

$\frac{2\sqrt{36x+1}}{2(6\sqrt{x})} = \frac{10}{8}$

$\frac{\sqrt{36x+1}}{6\sqrt{x}} = \frac{5}{4}$

Squaring both the sides,

$\frac{(\sqrt{36x+1})^2}{(6\sqrt{x})^2} = \frac{5^2}{4^2}$

$\frac{36x+1}{36x} = \frac{25}{16}$

$16(36x + 1) = 25 \times 36x$

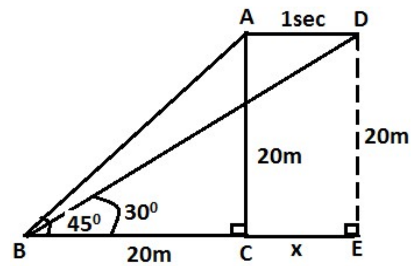
$576x + 16 = 900x$

$16 = 900x - 576x$

$16 = 324x$

$x = \frac{16}{324} = \frac{4}{81}$

(c) In Δ ABC,



$\tan 45^0 = \frac{AC}{BC}$

$1 = \frac{20}{BC}$

BC = <u>20m</u>

In Δ BDE,

$\tan 30^0 = \frac{DE}{BE}$

$\frac{1}{\sqrt{3}} = \frac{20}{BE}$

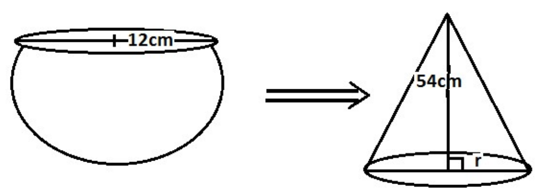
<u>BE = $20\sqrt{3}$</u> = <u>34.64 m</u>

CE = BE – BC = 34.64 – 20 = 14.64 m

Speed $= \frac{d}{t} = \frac{14.64}{1}$ = <u>14.64 m/s</u>

Answer 9.

(a) Volume of hemisphere = volume of cone



$\frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$

$2r^3 = r^2 h$

$2 \times 12 \times 12 \times 12 = r^2 \times 54$

$r^2 = 64$

= <u>8cm</u>

(b) let x $= \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

x + y $= \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$ ---- (i)

x – y $= \begin{bmatrix} 1 & -10 \\ -5 & 7 \end{bmatrix}$ ---- (ii)

adding (i) and (ii)

2x $= \begin{bmatrix} 5+1 & 2-10 \\ 1-5 & 3+7 \end{bmatrix}$

$2\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 6 & -8 \\ -4 & 10 \end{bmatrix}$

Since matrices are equal,

A = 3; b = -4; C = -2; d = 5

X $= \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$

Substituting x in eq. (i); x + y $= \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$

y $= \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix} - x$

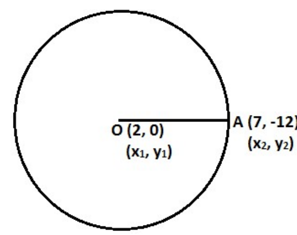
y $= \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ -2 & 5 \end{bmatrix}$

y $= \begin{bmatrix} 5-3 & 2(-4) \\ 1-(-2) & 3-5 \end{bmatrix}$

y $= \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$

ans: x $= \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$; y $= \begin{bmatrix} 2 & 6 \\ 3 & -2 \end{bmatrix}$

(c) i. r = AO



AO = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

AO = $\sqrt{(7 - 2)^2 + (-12 - 0)^2}$

AO = $\sqrt{(5)^2 + (-12)^2}$

AO = $\sqrt{25 + 144}$

AO = $\sqrt{169}$

AO = 13cm.

ii. Let the required point be A = [0, y]

Since it is equidistant from P & Q, it is a bisector of PQ:

Equation of PQ $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

$= \frac{4-8}{-4-0} = \frac{y-8}{x-0}$

$= \frac{-4}{-4} = \frac{y-8}{x}$

= -4x = -4(y – 8)

= -4x = -4y + 32

0 = 4x – 4y + 32

0 = x – y + 8 ……………………………(i)

m of PQ:-

x + 8 = y

m = 1

Thus, m × m' = -1

m' = -1

M is the mid-point of PQ

$\alpha = \frac{x_1 + x_2}{2}$, $\beta = \frac{y_1 + y_2}{2}$

$\alpha = \frac{-4 + 0}{2}$, $\beta = \frac{8 + 4}{2}$

$\alpha = \frac{-4}{2}$, $\beta = \frac{12}{2}$

$\underline{\alpha = -2}$ $\underline{\beta = 6}$

M = (-2, 6)

Answer 10.

(a) Scale (K) = 1 : 50

i. Ht. of model = Ht. of building X K

1 = h X $\frac{1}{50}$

h = 1 X 50 = $\underline{50m}$

ii. Area of model = K^2 X Area of building

$a^2 = \frac{1}{50} \times \frac{1}{50} \times 1600$

$a^2 = \frac{16}{25}$

a = $\frac{4}{5}$ m^2 = $\underline{0.8\ m^2}$

iii. Vol. of model = K^3 X volume of building

20 = $\frac{1}{50} \times \frac{1}{50} \times \frac{1}{50}$ X V

V = 20 X 50 X 50 X 50

= $\underline{25,00,000\ l}$

(b) Area of Δ = 6 square unit

AB = 3 units

BC = a units

A ΔABC = $\frac{1}{2}$ b X h

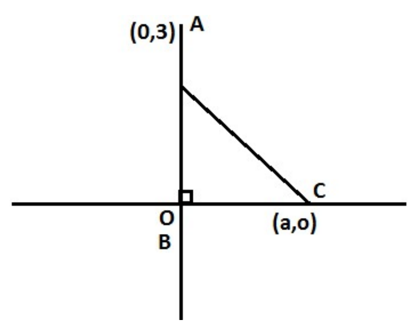
6 = $\frac{1}{2}$ X a X 3

$\frac{6 \times 2}{3}$ = a

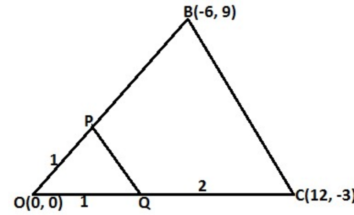
4 = a

C = a

0 = (4,0)



(c) In line OB, P = (α, β)



α $= \frac{m_1x_2 + m_2x_1}{m_1m_2}$, β = $\frac{m_1y_2 + m_2y_1}{m_1m_2}$

α $= \frac{1(-6) + 2(0)}{3}$, β = $\frac{1(9) + 2(0)}{3}$

α $= \frac{-6}{3}$, β = $\frac{9}{3}$

P = (α, β) = (-2, 3)

In line OC, Q = (α, β)

α $= \frac{m_1x_2 + m_2x_1}{m_1m_2}$, β = $\frac{m_1y_2 + m_2y_1}{m_1m_2}$

α $= \frac{1(12) + 2(0)}{3}$, β = $\frac{1(-3) + 2(0)}{3}$

α $= \frac{12}{3}$, β = $\frac{-3}{3}$

Q = (α, β) = (4, -1)

PQ = P = (-2, 3) = (x_1, y_1)

Q = (4, -1) = (x_2, y_2)

PQ $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

PQ $= \sqrt{(4 + 2)^2 + (-1 - 3)^2}$

PQ = $2\sqrt{13}$ units.

BC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

BC $= \sqrt{(12 + 6)^2 + (-3 - 9)^2}$

BC = $6\sqrt{13}$ units

Thus, $\frac{PQ}{BC} = \frac{2\sqrt{3}}{6\sqrt{3}}$

Thus, PQ = $\frac{1}{3}$BC, proved.

Answer 11.

(a) Let number of terms = n

First term (a) = 17

and common difference = 15 – 17

= –2

S_n = 72

∴ S_n $= \frac{n}{2}$ [2a + (n – 1)d]

72 $= \frac{n}{2}$ [2 × 17 + (n – 1) × (–2)]

72 × 2 = n [34 – 2n + 2]

144 = n(36 – 2n)

144 = 36n – 2n^2

2n^2 – 36n + 144 = 0

n^2 – 18n + 72 = 0

n^2 – 6n – 12n + 72 = 0

n(n – 6) – 12 (n – 6) = 0

(n – 6) (n – 12) = 0

Either n – 6 = 0, then n = 6

Or n = 12 = 0, then n = 12

∴ Number of terms = 6 or 12

(b) Let G_1, G_2 and G_3 be three means between

$\frac{1}{3}$ and 432, then

$\frac{1}{3}$, G_1, G_2, G_3, 432

6	1296
6	216
6	36
6	6
	1

Here, $T_1 = \frac{1}{3}$

$= a$

and $T_5 = ar^4 = 432$

$= 432$

$= \frac{1}{3}r^4$

$= r^4 = 432 \times 3 = 1296$

$= r^4 = 6^4$

$\therefore r = 6$

Now $G_1 = ar = \frac{1}{3} \times 6 = \underline{\underline{2}}$

$G_2 = ar^2 = \frac{1}{3} \times 6 \times 6 = \underline{\underline{12}}$

and $G_3 = ar^3 = \frac{1}{3} \times 6 \times 6 \times 6 = \underline{\underline{72}}$

(c)

Cl. Int.	**F**	x	**A = 57.5, t = x – A**	**i = 5, d = t/i**	**Fd**
40 – 45	5	42.5	– 15	– 3	– 15
45 – 50	12	47.5	– 10	– 2	– 24
50 – 55	20	52.5	– 5	– 1	– 20
55 – 60	16	57.5	0	0	0
60 – 65	10	62.5	5	1	10
65 – 70	8	67.5	10	2	16
70 – 75	5	72.5	15	3	15
75 – 80	4	77.5	20	4	16
Total	80				– 59 + 57 = – 2

$$\overline{x} = \left(\frac{\sum fd}{\sum f} \times i\right) + A$$

$$= \left(\frac{-2}{80} \times 5\right) + 57.5 = \frac{459}{8} = 57.375 = \underline{\underline{57.38}}$$

Answers of Practice Paper 14

Section I

Answer 1.

(a) A = $\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$; B = $\begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$

A^2 = B

$\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} (2\times2)+(12\times0) & (2\times12)+(12\times1) \\ (0\times2)+(0\times1) & (0\times12)+(1\times1) \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$

Since the matrices are equal, x = 36

(b) ∴ L.H.S $= \dfrac{\cos\theta\ \cot\theta}{1+\sin\theta}$

$= \dfrac{\cos\theta \cdot \frac{\cos\theta}{\sin\theta}}{1+\sin\theta}$

$= \dfrac{\cos^2\theta}{\sin\theta(1+\sin\theta)}$

$= \dfrac{1-\sin^2\theta}{\sin\theta\,(1+\sin\theta)}$

$= \dfrac{(1+\sin\theta)(1-\sin\theta)}{\sin\theta\,(1+\sin\theta)}$

$= \dfrac{1-\sin\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta}$

= cosec $\theta - 1$

R.H.S = cosec $\theta - 1$

L.H.S = R.H.S

Hence proved.

(c) $2x - \frac{5}{2} < x + \frac{3}{2} < 3x + \frac{11}{2} > x \epsilon R$

$2x - \frac{5}{2} < x + \frac{3}{2}$; $x + \frac{3}{2} < 3x + \frac{11}{2}$

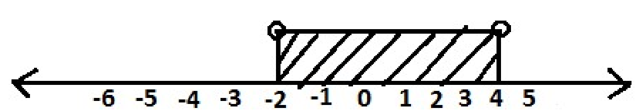
$\frac{4x-5}{2} < \frac{2x+3}{2}$; $x - 3x < \frac{11}{2} - \frac{3}{2}$

4x-5<2x=3 ; x - 3x < $\frac{8}{2}$

-5-3 < 2x-4x ; -2x < 4

4> x ; x > -2

4 > x > -2



Answer 2.

(a) $X^2 - 10x + 6 = 0$

a $= 1$

b $= -10$

c $= 6$

x $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

x $= \frac{-10 \pm \sqrt{102 - 40(6)}}{2(1)}$ $= \frac{10 \pm \sqrt{76}}{2}$ $= \frac{10 \pm 8.78}{2}$

x $= \frac{10 + 8.78}{2}$ or x $= \frac{10 - 8.78}{2}$

$= \frac{18.78}{2}$ or $= \frac{1.282}{2}$

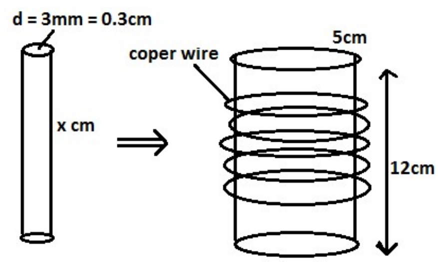
X $= 9.36$ or x $= 0.65$

X $= \underline{\{(9.36, 0.65)\}}$

(b) Circumference of cylinder $= 2\pi r$

$= 2 \times \frac{22}{7} \times 5$

$= \mathbf{\frac{220}{7}}$ **cm.**



ht. of 1 winding = diameter of wire

= 0.3cms

No. of turns $= \frac{12}{0.3}$ = 40 turns

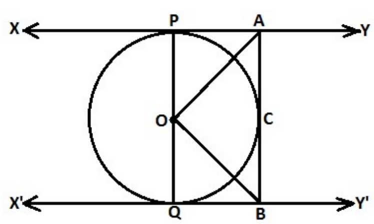
Length of wire $= n \times c$ $= 40 \times \frac{220}{7}$ = <u>1257.14 cms</u>

(c) Given: (i) xy II $x' y'$

(ii) AB is a tangent

TPT: L AOB = 90^0

Construction: Join OC



Statement	Reason
1. AP = AC	Tangents from a point outside the circle are equal.
2. OP = OC	Radii Of the same circle.
3.∠OPA = ∠ACO = 90^0	Tangents are ⊥ to the centre of the circle
4. ∠PAC = 90^0	Two tangents intersect each other at right angles
5. ∴ In square PACO, ∠POC = $360^0 - (90^0 + 90^0 + 90^0)$ = 90^0 ∴ Square PACO is a square	Sum of all the angles of a square are 560^0 Adjacent sides are equal and all angles are 90^0
6. ∴ ∠ POC = 90^0	Angle of a square
7. ∠ AOC = 45^0	Diagonal bisect the square

8. similarly OC BQ is also a square	
9. $\angle$ QOC = 90^0	Angle of a square
10. $\angle$COB = 45^0	Diagonal bisect the square
11. $\therefore$ $\angle$ AOB = $45^0 + 45^0 = 90^0$	Addition property

Answer 3.

(a) $\frac{7a+8b}{7c+8d} = \frac{7a-8b}{7c-8d}$

$\frac{7a+8b}{7a-8b} = \frac{7c+8b}{7c-8d}$ (by altemendo)

By componendo and dividendo,

$\frac{7a+8b+7a-8b}{7a+8b-7a+8b} = \frac{7c+8d+7c-8d}{7c+8b-7c+8d}$

$\frac{14a}{16b} = \frac{14c}{16d}$

$\frac{a}{b} = \frac{c}{d}$

Hence proved <u>a : b = c : d</u>

(b)

Express train	Ordinary train
S = x km/hr d = 240 km t = $\frac{d}{s}$ = $\frac{240}{x}$ hrs.	S = (x – 12) km/hr d = 240km t = $\frac{d}{s}$ = $\frac{240}{(x-12)}$ hrs.

Ordinary train takes 1hr. more than express train

$\therefore$ The equation is:

$\frac{240}{x-12} - \frac{240}{x} = 1$

$\frac{240x-240(x-12)}{x(x-12)} = 1$

240x – 240x + 2880 = $x^2 - 12x$

O = $x^2 - 12x - 2880$

O = $x^2 - 12x - 2880$

x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-12) \pm \sqrt{(-12)^2-4(1)(-880)}}{2(1)}$

= $\frac{12 \pm \sqrt{144 + 11520}}{2}$

= $\frac{12\pm\sqrt{11644}}{2}$

= $\frac{12 \pm 108}{2}$

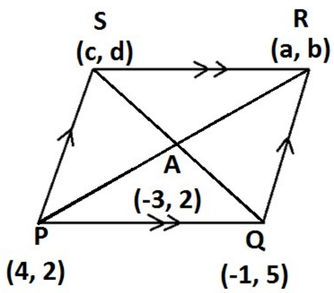
x = $\frac{12 + 108}{2}$ or

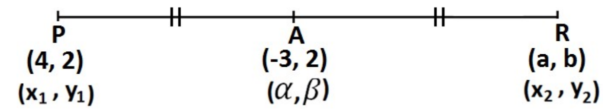
x = $\frac{12-108}{2}$

= 60 or -48

Since speed cannot be negative: <u>s = 60km/hr</u>

(c)





$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$-3 = \frac{4 + a}{2}$; $2 = \frac{2 + b}{2}$

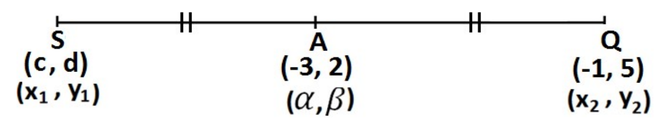
$-6 = 4 + a$; $4 = 2 + b$

$-a = 4 + 6$; $b = 4 - 2$

$-a = 10$; $b = 2$

$a = -10$

R (a, b) = <u>(– 10, 2)</u>



$\alpha = \frac{x_1 + x_2}{2}$; $\beta = \frac{y_1 + y_2}{2}$

$-3 = \frac{c - 1}{2}$; $2 = \frac{d + 5}{2}$

$-6 = c - 1$; $4 = d + 5$

$C = -6 + 1$; $d = 4 - 5$

$C = -5$; $d = -1$

S (c, d) = <u>(– 5, – 1)</u>

Answer 4.

(a) x = Rs.900

n = 2yrs = 24 months

r = 5 %

M.V =?

M.V = nx $= \frac{xrn\,(n+1)}{2400}$ $= (900 \times 24) + \frac{900 \times 5 \times 24 \times 25}{2400}$

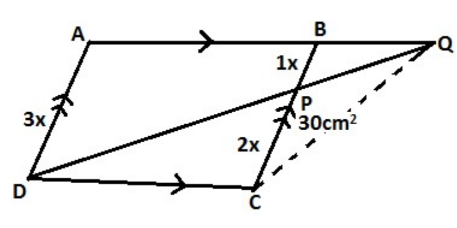
= 21600 + 1125 = <u>Rs. 22725</u>

(b) Given:

i. ABCD is a parallelogram.

ii. P is a point on BC.

iii. BP : PC = 1 : 2.



iv. DP produced meets AB at Q.

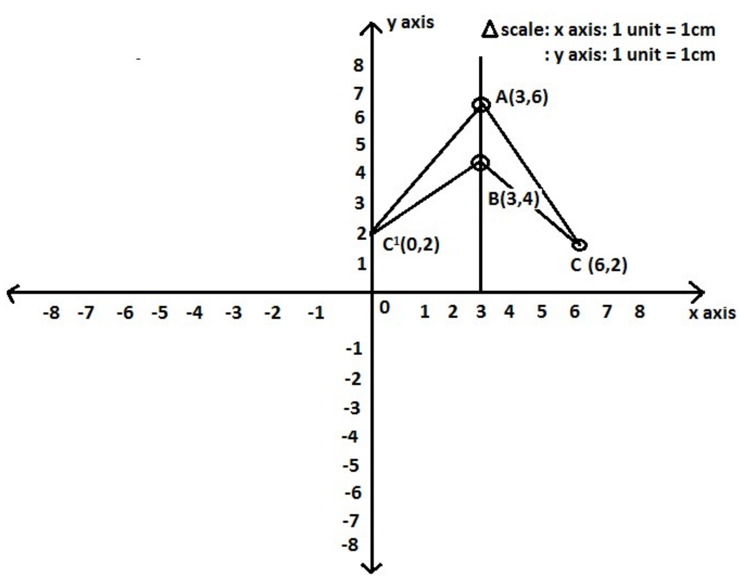
v. Area of ΔCPQ = 30cm^2

<u>To find:</u>

i. Area of ΔCDP. Ii. Area of parallelogram ABCD.

Statement	Reason
1. A Δ CPQ = 30cm^2	Given
2. $\frac{A\,\Delta\,BQP}{A\,\Delta\,QPC} = \frac{1}{2}$ $\frac{A\,\Delta\,BQP}{30} = \frac{1}{2}$ = 15 cm^2	If many Δs have the common vertex & their based are along the same straight line, the ratio of their areas is equal to the ratio between the lengths of their bases.
3. In Δ BPQ & ΔDPC, ∟ P = ∟P ∟P = ∟PCD ∴ Δ BPQ ~ Δ DPC	 Vertically opposite Int. alt angle AA Correspondence
4. $\frac{BP}{PC} = \frac{PQ}{PD} = \frac{BQ}{DC}$	Corresponding sides of similar Δs.
5. $\frac{A\,\Delta\,PBQ}{A\Delta\,CDP} = \frac{BP^2}{PC^2}$ $\frac{15}{A\,\Delta\,CDP} = \frac{1x^2}{(2x)^2}$ ∴ A Δ CDP = 60cms	Areas of similar Δs are proportional to the squarer of their corresponding sides.
6. In Δ QBP & Δ QAD LQ = LQ LQBP = L QAD ∴ Δ QBP ~ Δ QAD $\frac{QB}{QA} = \frac{QP}{QD} = \frac{BP}{AD}$	 Common L Corresponding L AA Corresponding Corresponding sides of similar Δs
7. $\frac{A\,\Delta\,QBP}{A\ trap.BPDA} = \frac{BP^2}{AD^2 - BP^2}$ $\frac{15}{A\ trap\ BPDA} = \frac{BP^2}{(3x^2) - x^2}$ $\frac{15}{A\ trap\ BPDA} = \frac{x^2}{8x^2}$ A tarp. BPDA = 120 cm^2 ∴ A parallelogram ABCD = A Δ CDP + A trap. BPDA = 60 + 120 = <u>180 cm^2</u>	

(c)



(i) Coordinates of C' = (0,2)

(ii) AC' BC arrow head

Area $= 2 \times \frac{1}{2} b \times h = 2 \times \frac{1}{2} x\ 2\ x\ 4 =$ 8 sq. units

(iii) The line x = 3 is the line of symmetry of the figure AC' BC.

Section II

Answer 5.

(a) Let x – 1 be a factor,

0 = x – 1

1 = x

f (1) = 0

f (x) $= 2x^3 + 3x^2\ 11x - 6$

f (1) $= 2(1)^3 + 3(1)^2 -11(1) -6 = 2 + 3 - 11 - 6 = 5 - 11 - 6 = -12$

f (1) is not a factor;

let x + 1 be a factor.

0 = x + 1

0 – 1 = x

f (-1) = 0

f (-1) $= 2(-1)^3 + 3(-1)^2 -11x - 6$

f (-1) $= 2(-1)^3 + 3(-1)^2 -11(-1) - 6 = -2 + 3 + 11 - 6 = -8 + 14 = 6$

f (-1) Is not a factor; let n -2 be a factor = 6

0 = x – 2

2 = x

f (x) $= 2x^3 + 3x^2 -11x -6$

f (2) $= 2(2)^3 + 3(2)^2 -11(2)-6 = 16 + 12 - 22 - 6 = 28 - 28 = 0$

x – 2 is a factor of $2x^3 + 3x^2 - 11x - 6$

$$
\begin{array}{r}
2x^2 + 7x + 1 \\
x-2 \overline{\big)\, 2x^3 + 3x^2 - 11x - 6} \\
2x^3 - 4x^2 \\
- \quad + \\
\hline
7x^2 - 11x \\
7x^2 - 14x \\
- \quad + \\
\hline
3x - 6 \\
\underline{3x - 6} \\
0
\end{array}
$$

f(x) $= (x - 2)(2x^2 + 7x + 3)$

$= (x - 2)(2x^2 + 6x + 1x + 3)$

$= (x - 2)[2x(x + 3) + 1\ (x + 3)]$

$= \underline{(x - 2)(x + 3)(2x + 1)}$

(b) f (x) $= x^3 + 10x^2 - 37x + 26$

Let $x - 1 = 0$

$\therefore x$ = 1

f (1) = $(1)^3 + 10\,(1)^2 - 37\,(1) + 26$

= 1 + 10 – 37 + 26 = 37 – 37 = $\underline{\underline{0}}$

Since f (1) = 0 ; $(x - 1)$ is a factor of f (x).

$$
\begin{array}{r|l}
 & x^2 + 11x - 26 \\
\hline
x - 1 & x^3 + 10x^2 - 37x + 26 \\
 & \underset{-}{+}x^3 \underset{+}{-} x^2 \\
\hline
 & \quad 11x^2 - 37x \\
 & \quad \underset{-}{+} 11x^2 \underset{+}{-} 11x \\
\hline
 & \qquad - 26x + 26 \\
 & \qquad \underset{+}{-} 26x \underset{-}{+} 26 \\
\hline
 & \qquad\qquad 0 \\
\hline
\end{array}
$$

f (x) = Divisor × Quotient

f (x) = $(x - 1)(x^2 + 11x - 26)$

= $(x - 1)\,(x^2 + 13x - 2x - 26)$

= $(x - 1)\,[\,x\,(x + 13) - 2\,(x + 13)]$

= $\underline{\underline{(x - 1)\,(x + 13)\,(x - 2)}}$

(c)

Investment I	Investment II
Fv = Rs 100 n = 60 shares r = 18 % Mv = Rs 160 (sp) Sales proceed = mv x h = 160 x 60 = Rs 9600 d = $\frac{Fvnr}{100}$ = $\frac{100\,X60\,X\,18}{100}$ = 1080	I = Rs 9600 Fv = Rs 50 n = ? Mv = $50 - \frac{4}{100}$ = Rs 48 r % = 18% n = $\frac{I}{mv}$ = $\frac{9600}{100}$ = 200 shares d = $\frac{fv\,X\,n\,X\,r}{100}$ = $\frac{50\,X200\,X18}{100}$ = Rs 1800

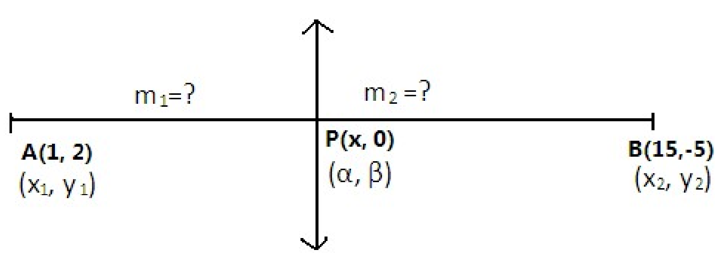
i. Sales proceed = Rs. 9600

ii. n = 200 shares

iii. Annual dividend from new shares = 1800

Answer 6.

(a)



$\beta = \frac{m_1\ y_2 +\ m_2\ y_1}{m_1 + m2}$

$0 = \frac{-5m_1 + 2m_2}{m_1 + m2}$

$0 = -5m_1 + 2m_2$

$5m_1 = 2m_2$

$\frac{m_1}{m_2} = \frac{2}{5}$

$m_1; m_2 = 2:5$

$\alpha = \frac{M_1 x_2 + M_2 x_1}{M_1 + M_2} = \frac{(2\ X\ 15) + (\ 5\ X\ 1)}{2+5} = \frac{30+5}{7}$

$= \frac{35}{7} = 5$

$P = (\alpha, \beta) = \underline{(5,0)}$

(b) Let the two numbers = a, b

Mean proportional. = 12

∴ the third proportional. = 96

The equation are:

$\frac{a}{b} = \frac{b}{96}$

$96a = b^2$

$96a = 12^2$

$a = \frac{12^2}{96}$

$a = 1.5$

∴ The no.s are: 1.5; 12; 96

(c) <u>EQ 1</u>: $3x - 2y = 5$

$3x - 5 = 2y$

$\frac{3x}{2} - \frac{5}{2} = y$

$m = \frac{3}{2}$

<u>EQ 2: </u>$2x + ky + 7 = 0$

$Ky = -2x - 7$

$y = \frac{-2x}{k} \frac{-7}{k}$

$M' = \frac{-2}{k}$

i. If lines are parallel to each other

$m = m'$

$\frac{3}{2} = \frac{-2}{K}$

$K = \frac{-2\ X\ 2}{3}$

$K = \frac{-4}{3}$

$K = -1\frac{1}{3}$

ii. If lines are ⊥ to each other

$Mm' = -1$

$\frac{3}{1}\ X \frac{-2}{k} = -1$

$\frac{-6}{2K}$ = -1

-6 = 2K

6 = 2K

3 = K

Answer 7.

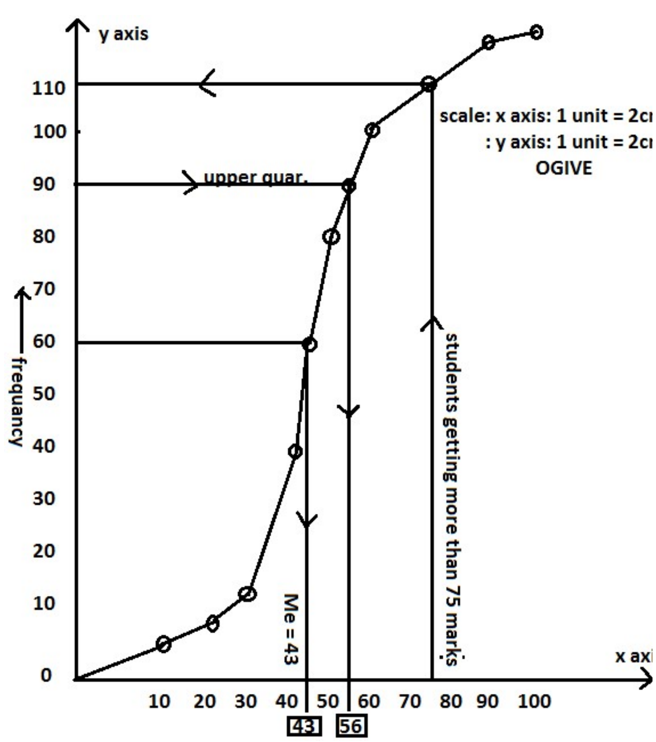
(a)

C.7	F	M	A= 17.5 D = m-a	I = 5 T = d/I	Ft
0-5	5	2.5	-15	-3	-15
5-10	8	7.5	-10	-2	-16
10-15	19	12.5	-5	-1	-19
15-20	25	17.5	0	0	0
20-25	27	22.5	5	1	27
25-30	20	27.5	10	2	40
30-35	10	32.5	15	3	30
35-40	6	37.5	20	4	24
Total	120				-50 +121=71

M $= A + \left[\frac{\sum ft}{\sum f}\right] x\, i$ $= 17.5 + \left[\frac{71}{120}\, x\, 8\right]$ = 17.5 x $\frac{71}{24}$

= 17.5 + 2.96 = <u>20. 96</u>

(b)



C 1	F	CF
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120
Total	120	-

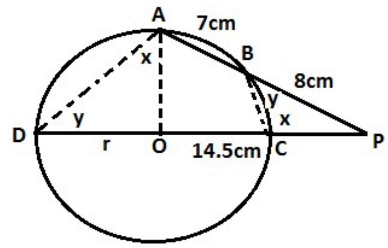
i. median = 62

ii. Q_3 = 82

iii. Above 75% = 123 – 113 = 10 students.

Answer 8.

(a) Given:



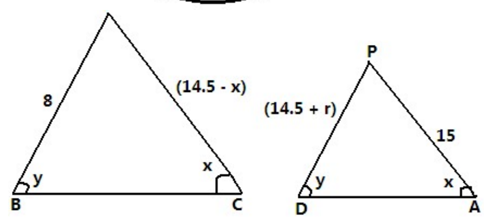
i. AB and DC are two chords.

ii. A circle with Centre O

iii. PB = 8cm

iv. BA = 7cm

v. OP = 14.5cm



<u>To find :</u> The radius of the circle.

Statement	Reason
1. PB = 8 cm	Given
2. AB = 7cm	
3. AP = PB + AB = 8 + 7 = 15cm	
4. OP = 14.5 cm	Given
5. In Δ PBC & Δ PAD i) L BCP = BAD ii) LPBC = LADP iii) Δ PBC ~ Δ PAD iv) $\frac{PB}{PD} = \frac{PC}{PA} = \frac{BC}{DA}$ v) $\frac{PB}{PD} = \frac{PC}{PA}$ $= \frac{8}{14.5 + R}$ $= \frac{14.5 - R}{15}$ 8 X 15 = $14.5^2 - R^2$ R^2 = $14.5^2 - 120$ = 210.25 – 120 = 90.25 R = 9.5 cm	Exterior angle is equal two interior opposite angle. exterior angle is equal two interior opposite angle AA correspondence Corresponding sides of similar triangle are proportional. Pythagoras theorem

(b) In an A.P.

First term a = –4

l = 29

S_n = 150

l = a + (n – 1)d

29 = –4 + (n – 1)d

(n – 1)d = 29 + 4 = 33

∴ (n – 1)d = 33

nd – d = 33

nd = 33 + d

n $= \frac{33 + d}{d}$

and S_n $= \frac{n}{2}[2a + (n - 1)d]$

150 $= \frac{n}{2}[2(-4) + 33]$

150×2 $= n[-8 + 33]$

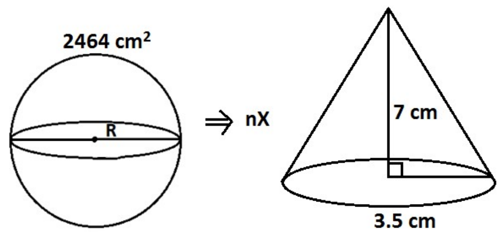
300 $= n(25)$

n $= \frac{300}{25} = 12$

and d $= \frac{33}{n-1} = \frac{33}{12-1} = \frac{33}{11} = \underline{\underline{3}}$

Hence d $= 3$

(c)



TSA $= 2464$

$4\pi R^2$ $= 2464$

$4 \times \frac{22}{7} \times R^2$ $= 2464$

R^2 $= \frac{2464 \times 7}{4 \times 22}$

R^2 $= 196$

R $= \underline{\underline{14\text{ cm}}}$

Vol. of sphere = n × vol. of cone

$\frac{4}{3}\pi R^3 = n \times \frac{1}{3}\pi R^2 h$

$4(R)^3 = n \times (3.5)^2 \times 7$

$4(14)3 = n \times 85.75$

$\frac{4 \times 2744}{85.75} = n$

$n = \underline{\underline{128}}$

Answer 9.

(a) 5 G.M.'s between 1 and 27

Let G_1, G_2, G_3, G_4, and G_5 be the 5 G.M.'s

$\therefore 1, G_1, G_2, G_3, G_4, G_5, 27$

Here, $T_1 = a$ $= 1$

$T_7 = ar^6$ $= 27$

Dividing, we get

$\frac{ar^6}{a} = \frac{27}{1}$

$r^6 = 27 = (\sqrt{3})^6$

$\therefore r = \sqrt{3}$

Now, G.P. will be

1, $\sqrt{3}$, 3, 3$\sqrt{3}$, 9, 9$\sqrt{3}$, 27

(b) Two points are given A (– 3, 7) and B (9, – 1)

∵ M is the mid – point of line joining AB.

∴ Co – ordinates of M will be = $\left(\frac{-3+9}{2}, \frac{7-1}{2}\right)$ or $\left(\frac{6}{2}, \frac{6}{2}\right)$ or (3, 3)

Let D (0, 0) be the origin and the point R (2, 2) divides OM in the ratio p:q

∴ x $= \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

2 $= \frac{p \times 0 + q \times 3}{p + q}$

2 (p + q) = 3q

2p + 2q = 3q

2p = 3q – 2q

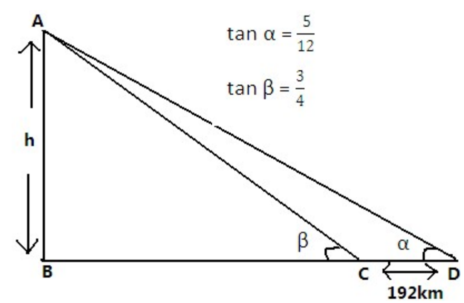
2p = q

∴ $\frac{p}{q} = \frac{1}{2}$

p : q = 1 : 2

(c) CD = h $[\tan(90° - \alpha) - \tan(90° - \beta))]$

CD = h $[\cot d - \cot \beta]$



192 = h $\left[\frac{12}{5} \frac{-4}{3}\right]$

192 = h $\left[\frac{36-20}{15}\right]$

192 = h $\frac{16}{15}$

$\frac{192 X 15}{16} = h$

<u>180m</u> = <u>h</u>

Answer 10.

(a) Let the given line 2x + y = 4 divides the line segment joining the points P (2, – 2) and Q (3, 7) in the ratio k : 1 at a point (x, y) on it.

∴ x $= \frac{kx_2 + 1.x_1}{k + 1}$

y $= \frac{ky_2 + 1.y_1}{k + 1}$

x $= \frac{k \times 3 + 1 \times 2}{k + 1} = \frac{3k + 2}{k + 1}$

and y $= \frac{7 \times k - 2 \times 1}{k + 1} = \frac{7k - 2}{k + 1}$

∴ This point lies on the given line.

∴ It is will satisfy it.

∴ $\frac{2(3k+2)}{k+1} + \frac{7k-2}{k+1} = 4$

$\frac{6k+4+7k-2}{k+1} = 4$

6k + 4 + 7k – 2 = 4k + 4

13k + 2 = 4k + 4

13k – 4k = 4 – 2

9k = 2

k = $\frac{2}{9}$

∴ Ratio is $\frac{2}{9}$ or 2 : 9

(b) Given:

i. DE ∥ BC ii. $\frac{AD}{DB} = \frac{3}{2}$

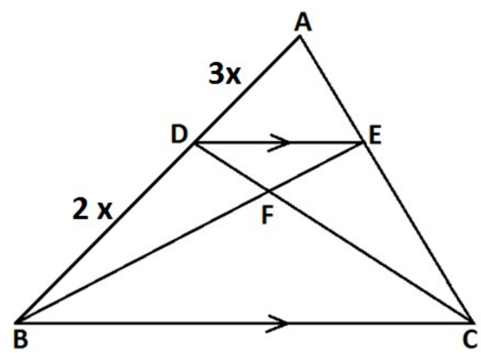
To find:

$\frac{AD}{AB}$

$\frac{DE}{BC}$

$\frac{EF}{FB}$

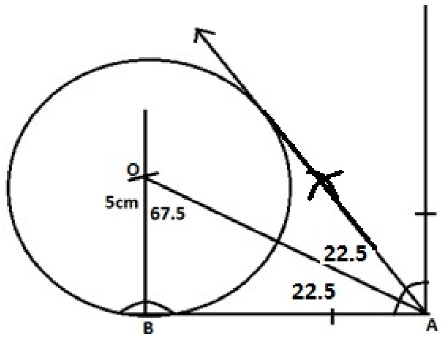
$\frac{A\,\Delta\,DEF}{A\,\Delta\,BFC}$



T . P. T : A ΔDEF ~ ΔCBF.

	Statement	Reasons
1.	In Δ ADE and Δ ABC, ∠DAE = ∠BAC ∠ADE = ∠ABC Δ ADE ~ Δ ABC	 Common angle Corresponding angles. By AA test of similarity.
2.	$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$	Corresponding sides of similar triangle are proportional.
3.	$\frac{AD}{AB} = \frac{AO}{AB+DB} = \frac{3}{3+2} = \frac{3}{5}$	
4.	$\frac{DE}{BC} = \frac{AD}{AB} = \frac{3}{5}$	From statement 2 above.
5.	In Δ DFE and ΔBFC, ∠ DFE = ∠ BFC ∠EDF = ∠FCB Δ DFE ~ Δ BFC	 Common angle Interior alternate angles. By AA test of similarity.
6.	$\frac{FD}{FC} = \frac{FE}{FB} = \frac{DE}{BC}$	Corresponding sides of similar triangles are proportional.
7.	∴ $\frac{FE}{FB} = \frac{DE}{BC} = \frac{3}{5}$	From statement 4 above.
8.	$\frac{A\,\Delta DEF}{A\,\Delta\,BFC} = \frac{(DE)^2}{(BC)^2} = \frac{(3)^2}{(5)^2} = \frac{9}{25}$	Areas of similar triangles are proportional to the squares on their corresponding sides.

(c)



Answer 11.

(a) Given:

i. AB ∥ DC

ii. ∠BCE = 80°

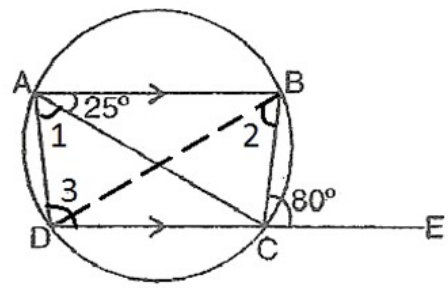
iii. ∠BAC = 25°

To find:

i. ∠CAD

ii. ∠CBD

iii. ∠ADC



Cons: Join BD.

	Statement	Reason
1.	∠CAB = 25°	Given.
2.	∠BCE = 80°	Given.
3.	∠BAD = ∠BCE = 80°	The exterior angle of a cyclic quadrilateral ABCD is equal to the interior opposite angles.
4.	∠CAD + ∠BAC = ∠BAD ∠CAD + 25° = 80° ∠CAD = 80° – 25° = 55°	By addition property.
5.	∠CBD = ∠CAD = 55°	Angle in the same segment are equal.
6.	∠ADC + ∠BAD = 180° ∠ADC + 80° = 180° ∠ADC = 180° – 80° = 100°	Co – interior angles. Given AB ∥ DE.

(b) ∴ L.H.S $= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \frac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)} = \frac{\tan A\,(1 - \sin^2 A - \sin^2 A)}{(\cos^2 A + \cos^2 A - 1)}$

$= \frac{\tan A\,(\cos^2 A - \sin^2 A)}{(1 - \sin^2 A + \cos^2 A - 1)}$

$= \frac{\tan A\,(\cos^2 A - \sin^2 A)}{(\cos^2 A + \sin^2 A)}$

= tan A × $\frac{1}{1}$

= tan A

= R.H.S

= tan A

= L.H.S = R.H.S

= Hence proved.

(c) Given:

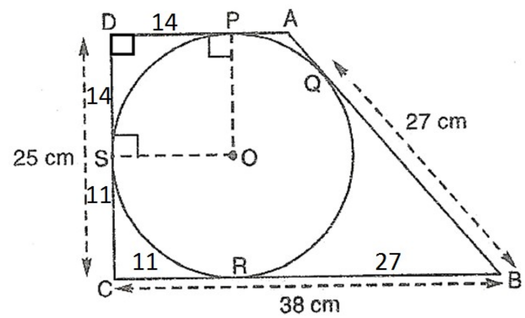
i. BC = 38 cm

ii. QB = 27 cm

iii. DC = 25 cm

iv. AD ⊥ DC

To find : Radius of the circle.



	Statement		Reason
1.	BC	= 38 cm	Given.
2.	QB	= 27 cm	Given.
3.	DC	= 25 cm	Given.
4.	BQ	= BR = 27 cm	Tangents drawn from any point outside the circle are equal.
5.	CR	= CB – RB = 38 – 27 = 11 cm	By addition property.
6.	CS	= CR = 11 cm	Tangent drawn to the circle from any point outside the circle are equal.
7.	DS	= DC – SC = 25 – 11 = 14 cm	By addition property.
8.	DP	= DS = 14 cm	Tangent drawn to the circle from any point outside the circle are equal.
9.	∠OSD	= ∠OPD = 90°	Tangents are ⊥ to the radius at the point of contact.
10.	∠SDP	= 90°	Given
11.	∠DPO + ∠POS + ∠DSO + ∠PDS 90° + 90° + 90° + ∠POS ∠POS	= 360° = 360° = 360° – 270° = 90°	Sum of the angles of a quadrilateral DPOS is 360°.
12.	OP	= PS	Radii of the same circle.
13.	Thus DPOS is a square.		2 pairs of adjacent sides are equal each equal to 90°. From statement no. 8, 11, 12 above.
14.	PO = OS = SD = DP Radius	= 14 cm = 14 cm	Sides of a square.

Answers of Practice Paper 15

Section I

Answer 1.

(a) X $= \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$

I $= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

TPT: $6X - X^2 = 9I$

LHS $= 6X - X^2$ $= 6\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -1 & 2 \end{bmatrix}^2$

$= \begin{bmatrix} 24 & 6 \\ -6 & 12 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$ $= \begin{bmatrix} 9 & 9 \\ 0 & 0 \end{bmatrix}$

RHS $= 9I$

$9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

LHS = RHS

(b) Possible outcomes:

i. Doublet m = 6

n = 36

P(a doublet) $= \frac{m}{n}$

P(a doublet) $= \frac{6}{36}$

P(a doublet) $= \frac{1}{6}$

ii. sum of 5

m = 4

n = 36

P(a sum of 5) $= \frac{m}{n}$

P(a sum of 5) $= \frac{4}{36}$

P(a sum of 5) $= \frac{1}{4}$

iii. Both are even

m = 9

n = 36

P(both even number) $= \frac{m}{n}$

P(both even number) $= \frac{1}{4}$

(c) $3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$, x ∈ I

$3 \geq \frac{x-4}{2} + \frac{x}{3}$

$3 \geq \frac{3x-12+2}{6}$ $= \frac{30}{5} \geq x$ $= 6 \geq x$

$= \frac{x-4}{2} + \frac{x}{3} \geq 2$ $= \frac{3x-12+2x}{6} \geq 2$

$= x \geq \frac{24}{5}$ $= 4.8$

$6 \geq x \geq 4.8$; x ∈ I

x = {5, 6}

Answer 2.

(a) Let $\frac{a}{b} = \frac{b}{c} = k$

a = bk

b = ck

a = ck(k)

b = ck

a = ck^2

b = ck

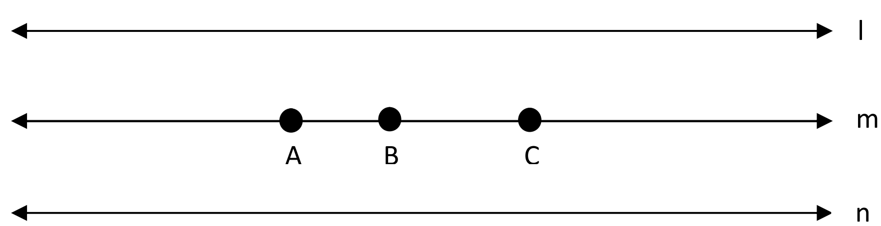
$= \frac{a}{c} = \frac{a^2}{b^2}$

LHS: $\frac{a}{c} = \frac{ck^2}{c} = k^2$

RHS: $\frac{a^2}{b^2} = \frac{c^2k^2}{c^2k^2} = k^2$

Hence Proved.

(b) Locus of points which are equidistant from 3 distinct points on a line is anywhere on the path around the segment formed by the 3 equidistant point.



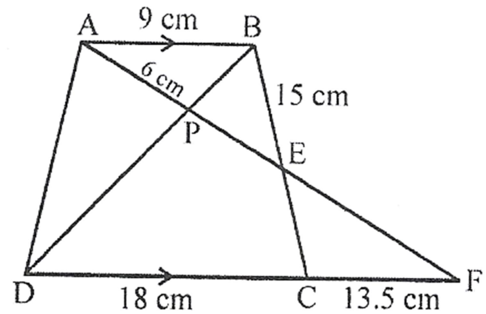
Locus is anywhere on line l and n which is parallel to line m and moving at a fixed distance on either side.

(c) Given:

i. AB = 9 cm
ii. DC = 18 cm
iii. CF = 13.5 cm
iv. AP = 6 cm
v. BE = 15 cm

To find:

i. EC
ii. AF



Statement		Reasons
1.	In ΔAEB and ΔECF ∠AEB = ∠CEF ∠ABE = ∠ ECF ∴ Δ AEB ~ Δ ECF	 Vertically opposite angles. Alternate angles. AA axiom
i.	$\frac{AB}{CF} = \frac{BE}{EC}$ $= \frac{9}{13.5}$ $= \frac{15}{EC}$ EC $= \frac{15 \times 13.5}{9}$ = <u>22.5 cm</u>	
ii.	Similarly ΔAPB and ΔDPF ∠APB = ∠DPF ∠ABP = ∠ PDE ∴ Δ APB ~ ΔDPF ∴ $\frac{AB}{DF} = \frac{AP}{PF}$ $\frac{9}{18 + 13.5} = \frac{6}{PF}$ $\frac{9}{31.5} = \frac{6}{PF}$ PF $= \frac{6 \times 31.5}{9}$ = 21 cm ∴ AF = AP + PF = 6 + 21 = <u>27cm</u>	 Vertically opposite angles. Alternate angles. AA axiom.

Answer 3.

(a) A = (3, -4) = (x_1, y_1)

B = (5, -6) = (x_2, y_2)

Slope of AB = m = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(-4)}{5-3}$ = -1

Slope of <u>I</u> line

m X m^1 = -1

-1 X m^{-1} = -1

m^{-1} = 1

C is the midpoint of AB

$\alpha = \frac{3+5}{2} = 4$

$\beta = \frac{-4-6}{2} = -5$

C = (4, -5) = (x_1, y_1)

m = 1

$y - y_1 = m(x - x_1)$

$-x + y = -9$

$x - y = 9.$

(b) $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$

Using componendo – dividendo

we get ; $\frac{x+1}{x-1} = \frac{2\sqrt{a+2b}}{2\sqrt{a-2b}}$

$\frac{x+1}{x-1} = \frac{\sqrt{a+2b}}{\sqrt{a-2b}}$

On squaring both the sides

$\left(\frac{x+1}{x-1}\right)^2 = \left(\frac{\sqrt{a+2b}}{\sqrt{a-2b}}\right)^2$

$\frac{x^2+2x+1}{x^2-2x+1} = \frac{a+2b}{a-2b}$

Using CD again

$\frac{2x^2+2}{4x} = \frac{2a}{4b}$

$\frac{x^2+1}{2x} = \frac{a}{2b}$

$\frac{x^2+1}{x} = \frac{a}{b}$

$bx^2 + b = ax$

$bx^2 - ax + b = 0$

<u>Hence proved.</u>

(c) FV = Rs 80

n = 350 shares

r = 15%

MV = 80 – (10% of 80) = 72

I =?

D =?

R =?

I = Mv × n = 72 × 350 = 25200

D $= \frac{FV \times n \times r}{100} = \frac{80 \times 350 \times 15}{100} =$ Rs 4200

5 Profit = $\frac{4200 \times 100}{25200}$ = 16.66%

Answer 4.

(a) L.H.S = $\sin^4 A - \cos^4 A$

$= (\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A)$

$= (\sin^2 A - \cos^2 A)(1)$

$= \sin^2 A - \cos^2 A$

$= \sin^2 A - (1 - \sin^2 A)$

$= 2\sin^2 A - 1$

R.H.S = $2\sin^2 A - 1$

L.H.S = R.H.S

Hence proved.

(b) O $= (0, 1) = (x_{1,} y_1)$

P $= (5, -3) = (x_{2,} y_2)$

R $= (x, 6)$

$\propto = \frac{x_1 + x_2}{2}$

$0 = \frac{5+x}{2}$

= x

= -5

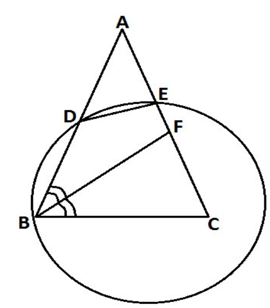
OR $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

OR $= \sqrt{(-5 - a)^2 + (6 - 1)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$ units

PR $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

PR $= \sqrt{(-5 + 5)^2 + (6 + 3)^2} = \sqrt{0 + 81} = \sqrt{81} = 9$ units

(c) Given:



i. ABC is an isosceles triangle such that AB = AC.

ii. A circle with Centre C

iii. CB as radius

iv. AB and AC cuts at D and E respectively.

To prove That: DE to the bisector BF of ∠ABC.

Statement	Reason
(i) ∠DBF = ∠FBC	BF is the bisector of ∠ABC
(ii) ∠ABC = ∠ACB	ΔABC is an isosceles Δ.
(iii) In Δ ABC, ∠BAC = $180^0 - 4X$	Sum of all angles of a triangle is 180^0.
(iv) In ΔABF, ∠AFB = $180^0 - [(180^0 - 4x) + x] = 3x$	Angle sum property
(v) ∠BDE = ½ Reflex ∠BCE = ½ $(360^0 - 2x) = 180^0 - x$	Reflex angle property,

∠ADE = $180^0 - (180^0 - x) = x$; ∠ADE = ∠ABF = x	Straight line angle and from (i)
(vi) In Δ AED, ∠AED = $180^0 - [(180^0 - 4x) + x] = 3x$	Angle sum property
(vii) ∠AED = ∠AFB = 3x DE||BF.	(ii) ; (i) and (ii) as it forms corresponding angles

Section II

Answer 5.

(a)

Category	Wages / day x	No. of worker (f)	fx
A	50	2	100
B	60	4	240
C	70	8	560
D	80	12	960
E	90	10	900
F	100	6	600
Total		42	3360

$\bar{x} = \frac{\sum fx}{\sum f} = \frac{3360}{42}$

= Rs. 80 per day

i. If number of workers are doubled frequency is doubled and hence there is no effect on mean value.

$\bar{x}$ = Rs. 80 per day

ii. If wages per day are increased by 60%,

New $\bar{x} = 80 + (\frac{60}{100} \times 80) = 80 + 48$

= Rs. 128 per day

iii. If number of workers in each category is doubled, no effect on $\bar{x}$.

If wages per day are reduced by 40%,

$\bar{x} = 80 - \left(\frac{40}{100} \times 80\right) = 80 - 32$ = Rs. 48/day

(b) $2x - \frac{1}{x} = 7$

$\therefore 2x^2 - 7x - 1 = 0$

a = 2

b = -7

c = -1

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{49 + 8}}{4} = \frac{7 \pm \sqrt{57}}{4}$

Thus,

$x = \frac{7+7.550}{4}$; $x = \frac{7-7.550}{4}$

$x = \frac{14.550}{4}$; $x = \frac{-0.550}{4}$

x = 3.6375 ; x = -0.1375

x = 3.64 or x = -0.14

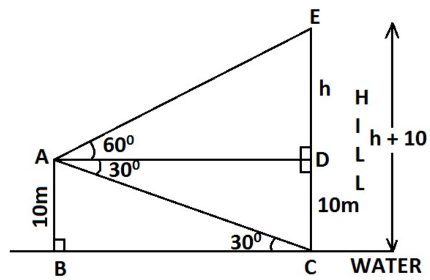
∴ SS(x) = {3.64, -0.14}

(c) In Δ ADE,

$\tan 60° = \frac{ED}{DA}$

$\sqrt{3} = \frac{h}{x}$

$h = \sqrt{3}\,x$ ---------- (i)



In Δ ABC,

$\tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{10}{x}$

$x = 10\sqrt{3}$ ----------- (ii)

⟹ substituting in (i)

$h = \sqrt{3}\,x = \sqrt{3} \times 10\sqrt{3} = 30$

Total height = h + 10 = 30 + 10 = 40 m

$x = \frac{h}{\sqrt{3}} = \frac{\sqrt{3}h}{3} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$ = 17.32m

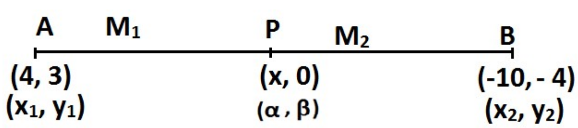
Answer 6.

(a)

Class	F	M	A = 175 , m – A	D = 10	Fd
150 – 160	6	155	-20	-2	-12
160 – 170	13	165	-15	-1	-13
170 – 180	15	175	0	0	0
180 – 190	8	185	15	1	8
190 – 200	8	195	20	2	16
	50				-25 + 24 = -1

$\overline{X} = A + \left(\frac{\epsilon fd}{\epsilon f} \; X \; i\right) = 175 + \left(\frac{-1}{50} \; X \; 10\right) = 175 - 0.2 = 179.8$

(b)



$\alpha = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$; $\beta = \frac{y_1 m_2 + m_1 y_2}{m_1 + m_2}$

$\alpha = \frac{3(-10) + 4(4)}{3 + 4}$; $0 = \frac{m_1(-4) + m_2(3)}{m_1 + m_2}$

$\alpha = \frac{-30 + 16}{7}$; $4m_1 = 3m_2$

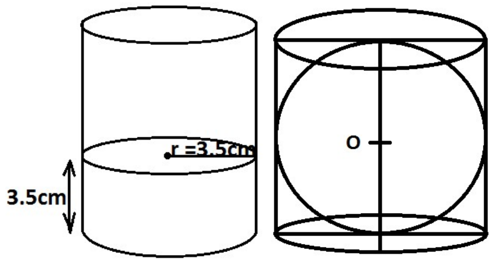
$\alpha = \frac{-30 + 16}{7}$; $4m_1 = 3m_2$

$\alpha = \frac{-14}{7}$; $= \frac{m_1}{m_2} = \frac{3}{4}$

$\alpha = \underline{\underline{-2}}$; $m_1 : m_2 = \underline{\underline{3 : 4}}$

$\therefore P(x, 0) = \underline{\underline{(-2, 0)}}$

(c)



i. T. S. A. of can in contact with water = C.S.A. of cylinder + Base area

$= 2\pi rh + \pi r^2$

$= \pi r(2h + r)$

$= \frac{22}{7} \times \frac{7}{2}(2 \times 7 + \frac{7}{2})$ $= 11(14 + 3.5)$

$= 11 \times 17.5$ $= \underline{\underline{192.5\text{ cm}^2}}$

ii. Sphere is removed,

Volume of water = Cylinder – Sphere

$= \pi r^2 h - \frac{4}{3}\pi r^3$

$= \pi r^2 (h - \frac{4r}{3}) = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \left(7 \times \frac{4}{3} \times \frac{7}{2}\right) = \frac{539}{6}\text{ cm}^3$

$= \underline{\underline{89.833\text{ cm}^3}}$

Volume of water = Volume of cylinder

$\frac{539}{6} = \pi r^2 h_1$

$\frac{539}{6} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h_1$

$h_1 = \frac{7}{3}\text{ cm} = 2\frac{1}{3}\text{ cm}$

Answer 7.

(a) let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$a = bk$

$c = dk$

$e = fk$

LHS $= \left(\frac{a^2b^2 + c^2d^2 + e^2f^2}{ab^3 + cd^3 + ef^3}\right)^3$ $= \left(\frac{b^2k^2b^2 + d^2k^2d^2 + f^2k^2f^2}{bkb^3 + dkd^3 + fkf^3}\right)^3$

$= \left(\frac{k^2b^4 + k^2d^4 + k^2f^4}{kb^4 + kd^4 + kf^4}\right)^3$ $= \left(\frac{k^2(b^4 + d^4 + f^4)}{k(b^4 + d^4 + f^4)}\right)^3$

$= \left(\frac{k^2}{k}\right)^3 \qquad = k^3$

RHS $= \frac{ace}{bdf} = \frac{bk \; X \; dk \; X \; fk}{bdf} = k^3$

LHS = RHS

Proved.

(b) Avg. of the given numbers $= \frac{1+2+2+3+3+3+4+4+4+4}{10} = \frac{30}{10} = 3$

Total number of events (n) = 10

No. of favourable outcome (m) = 3

Probability of getting's 3, P(ε) $= \frac{m}{n} = \frac{3}{10}$

(c) Given:

i. A circle is inscribed in the quadrilateral ABCD.

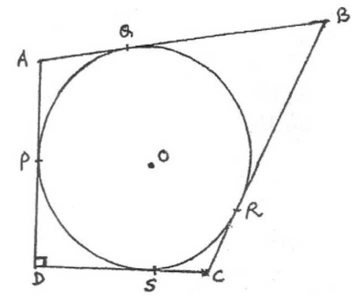
ii. BC = 38cm

iii. QB = 27cm

iv. DC = 25cm

v. ∠ADC = 90^0.

To Find: The radius of the circle.



Statement	Reason
(1) QB = 27cm	Given
(2) BQ = BR = 27	Tangents from the point outside the circle are equal.
(3) BC = 38 ; BR = 27 ; BC = BR + RC ; RC = 11cm	
(4) RC = SC ; SC = 11cm	Intersecting tangents of the same circle
(5) DS = DC – SC = 25 – 11 = 14cm	Addition property
(6) ∠OSD = 90^0 ; ∠OPD = 90^0	Angle between tangent and the center is always 90^0
(7) ∠PDS = 90^0	Given
(8) In Figure OPDS: ∠OPD = ∠OSD = ∠PDS = 90^0 ; ∠POS = 90^0	From 6 and 7 above and Angles of a quadrilateral is 360^0.
(9) OP = OS	Radii pof same circle
(10) DP[= DS	Tangents from a point outside the circle are equal.
(11) PDSO is a square	All angles 90^0 and 2 pairs of adjacent angles are equal.
(12) PD = SD = OP = OS = 11cm	From Statement 11.
(13) Radius = 11cm	

Answer 8.

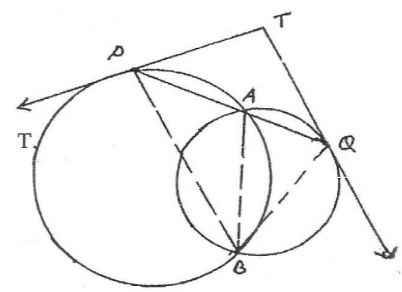
(a) Given:

i. Two circles intersect each other at A and B.

ii. Straight line PAQ through A meets the circles at P and Q.

iii. Tangents at P and Q meet at the point T.

To Prove: ∠PTQ + ∠PBQ = 180^0.



Statement	Reason
(1) TP = TQ	Tangents from a point outside the circle are equal
(2)Let ∠TPQ = ∠TQP = x	Base angles of isosceles triangle
(3)∠PTQ = 180^0 – 2x	Angle sum property
(4) ∠PBA = ∠TPA = X	Alternate segment theorem
(5) ∠TQP = ∠QBA = x	Alternate segment theorem
(6)∠PBQ = x + x = 2x	Addition property
(7)∠PTQ + ∠PBQ = 180^0 – 2x + 2X = 180^0	Proved

(b) FV = Rs 25

n =?

r = 12%

MV = Rs. 33

I =?

D = Rs720

720 = $\frac{25 \times n \times 12}{100}$

n = 240 shares

I = MV × n = 240 × 36

Rs. = 8640

FV × r = R × 36

R = 8.33%

(c)

	Women	Son
5 years ago	X^2	X
Today	$X^2 + 5$	X + 5
Ten years later	$X^2 + 15$	X + 15

$X^2 + 15 = 2(x + 15) = x^2 – 2x + 15 – 30 = 0$

(x + 3) (x + 5) = 0 x = -3 or 5

ignoring the –ve sign we get x = 5 years

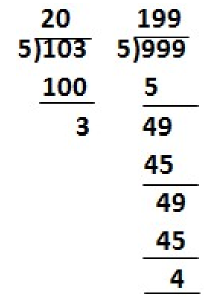
Sons age = x = 5 years

Women's present age = $x^2 + 15$ = 40years.

Answer 9.

(a) 3 digits numbers are 100, 101, 102, ………., 999

Number when divided by 5, remainder = 3



∴ 103, 108, 113, 118, ……., 998

First term (a) = 103

d = 108 – 103 = 5

l = 998

∴ l = a + (n – 1)d

998 = 103 + (n – 1) × 5

998 – 103 = (n – 1) × 5

$\frac{895}{5}$ = n – 1

179 = n – 1

n = 179 + 1 = 180

∴ S_n = $\frac{n}{2}$ [2a + (n – 1)d]

= $\frac{180}{2}$ [2 × 103 + (180 – 1) × 5] = 90 [206 + 179 × 5]

= 90 [206 + 895] = 90 × 1101 = <u>99090</u>

(b) Given:

i. AB, CD, EF are parallel lines.

ii. AB = 6cm

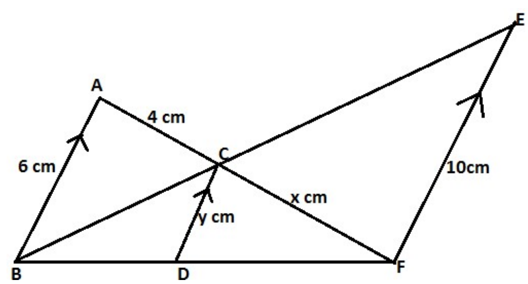
iii. CD = y cm

iv. EF = 10 cm

v. AC = 4 cm

vi. CF = x cm.

<u>To find</u> : The values of CF and CD.



Statement.	Reason.
1. In ΔABC = ΔCEF, i. ∠ACB = ∠ECF ii. ∠CAB = ∠CFE thus, ΔABC ≅ ΔCEF	 Vertically opposite angles. Interior alternate angles. By AA postulate.
2. $\frac{AB}{FE} = \frac{AC}{CF} = \frac{BC}{CE}$ Thus, $\frac{AB}{FE} = \frac{AC}{CF}$ $\frac{6}{10} = \frac{4}{x}$ x $= \frac{4 \times 10}{6}$ = 6.666cm = <u>6.67cm.</u>	By BPT.
3. In ΔFCD and ΔFAB, i. ∠F = ∠F ii. ∠FCD = ∠FAB thus, ΔFCD ≅ ΔFAB	 Common angle. Corresponding angles. By AA postulate.
4. $\frac{FC}{FA} = \frac{CD}{AB} = \frac{FD}{FB}$ $\frac{x}{x+4} = \frac{4}{6}$	By BPT. (contd....)

$\frac{40 \times 6}{6\left(4+\frac{40}{6}\right)} = \frac{y}{6}$ $\underline{y = 3.75\text{cm}}$	

(c) f (x) $= 2x^3 - 9x^2 - 11x + 30$

Let $x + 2 = 0$

$x = -2$

f $(-2) = 2(-2)^3 - 9(-2)^2 - 11(-2) + 30$

$= 2(-8) - 9(4) - 11(-2) + 30$

$= -16 - 36 + 22 + 30 \quad = \quad 52 - 52 \quad = \quad 0$

$\because$ f $(-2) = 0$, $(x + 2)$ is a factor of f (x)

$$
\begin{array}{r|l}
 & 2x^2 - 13x + 15 \\
\hline
x+2 & 2x^3 - 9x^2 - 11x + 30 \\
 & -(2x^3 + 4x^2) \\
\hline
 & -13x^2 - 11x \\
 & -(-13x^2 - 26x) \\
\hline
 & 15x + 30 \\
 & -(15x + 30) \\
\hline
 & 0
\end{array}
$$

f (x) = Divisor $\times$ Quotient

$= (x + 2)(2x^2 - 10x - 3x + 15)$

$= (x + 2)(2x(x - 5) - 3(x - 5))$

$= \underline{\underline{(x + 2)(2x - 3)(x - 5)}}$

Answer 10.

(a) In G.P.

$S_\infty = \frac{80}{9}$

Common ratio = $\frac{-4}{5}$

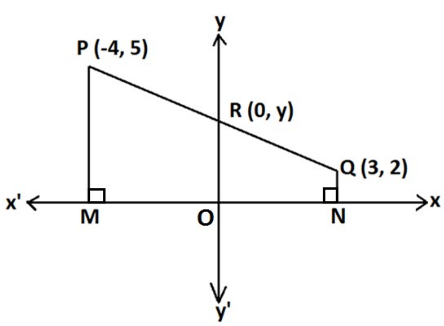
$S_\infty = \frac{80}{9}$

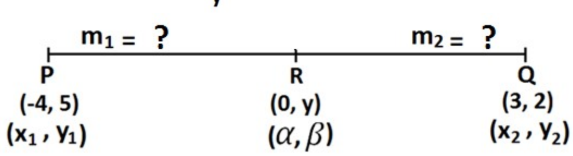
$= \frac{a}{1 - r} = \frac{80}{9} \qquad = \frac{a}{1 + \frac{4}{5}} = \frac{80}{9} \qquad = \frac{a}{\frac{9}{5}} = \frac{80}{9}$

$= a = \frac{80}{9} \times \frac{9}{5} \qquad = 16$

$= \quad T_1$ or a $\quad = \quad 16$

(b)





i. $\alpha = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$0 = \frac{(m_1)(3) + (m_2)(-4)}{m_1 + m_2}$

$0 = 3m_1 - 4m_2$

$4m_2 = 3m_1$

$\frac{M_1}{m_2} = \frac{4}{3}$

$m_1 : m_2 = \underline{\underline{4 : 3}}$

ii. $\beta = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\beta = \frac{(4 \times 2) + (3 \times 5)}{4 + 3}$

$\beta = \frac{8 + 15}{7}$

$\beta = \frac{23}{7}$

$\therefore R(\alpha, \beta) = \left(0, \frac{23}{7}\right)$

iii. A of trapezium PMNQ $= \frac{1}{2} \times$ (sum of II sides) $\times$ h

$= \frac{1}{2}(5 + 2) \times 7 \qquad = \frac{1}{2} \times 7 \times 7 \qquad = \underline{\underline{24.5 \text{ unit}^2}}$

(c)

Marks	**No. Of students**	x	**f** x
0 – 10	4	5	20
10 – 20	A	15	15a
20 – 30	20	25	500
30 – 40	B	35	35b
40 – 50	4	45	180
Total	28 + a + b		700 + 15a + 35b

Σf = 50

28 + a + b = 50

a + b = 22 ……………………………..(i)

$x = \frac{\Sigma fx}{\Sigma f}$

$23 = \frac{700 + 15a + 35b}{50}$

1150 = 700 + 15a + 35b

15a + 35b = 450

3a + 7b = 90 ……………………………..(ii)

⟹ Multiplying (i) with 3,

3a + 3b = 66……………………………..(iii)

⟹ subtract (iii) from (ii),

$$\begin{array}{r} 3a + 7b = 90 \\ \underline{-3a \pm 3b = -66} \\ 4b = 24 \end{array}$$

b = 6

⟹ substituting,

a + b = 22

a = 16

Answer 11.

a. ∴ L.H.S = $\sqrt{\frac{1+\sin A}{1-\text{si}}}$

$= \sqrt{\frac{(1+\sin A)}{(1-\sin A)}} \times \sqrt{\frac{(1+\sin A)}{(1+\sin A)}}$ $= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$ $= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

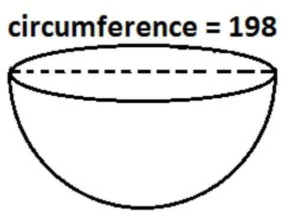
$= \frac{1+\sin A}{\cos A}$ $= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$ = sec A + tan A

R.H.S = sec A + tan A

L.H.S = R.H.S

Hence proved

(b)



$2\pi r$ = 198 cm

$2 \times \frac{22}{7} \times r$ = 198

r $= \frac{198 \times 7}{22 \times 2}$

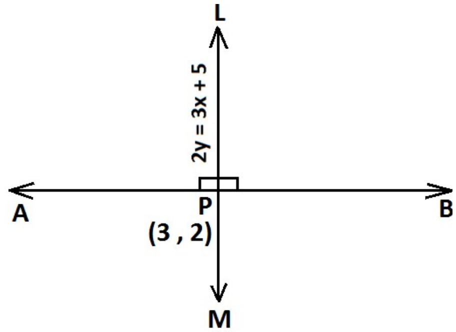
r $= \frac{63}{2}$ = 31.5 cm

Volume $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{63}{2} \times \frac{63}{2} \times \frac{63}{2}$ $= \frac{130977}{2}$ = 65,488.5 cm^3

(c)

i.



<u>m of line LM</u>:

$2y = 3x + 5$

$y = \frac{3}{2}x + \frac{5}{2}$

$m = \frac{3}{2}$

∴ m' of AB $= \left(\frac{-1}{m\ of\ LM}\right)$, perpendicular $= \left(\frac{-1}{3/2}\right) = \frac{-2}{3}$

Equation of line AB:

$m = \frac{-2}{3}$

$P = (3, 2) = (x_1, y_1)$

$m = \frac{y - y_1}{x - x_1}$

$\frac{-2}{3} = \frac{(y) - (2)}{(x) - (3)}$

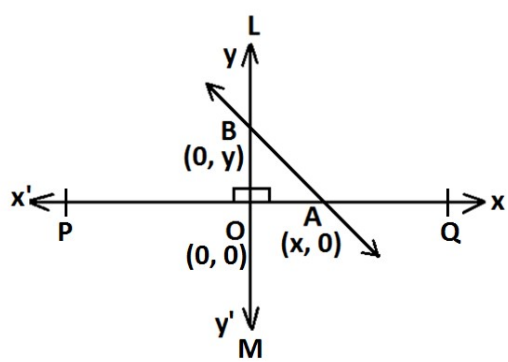
$\frac{-2}{3} = \frac{y - 2}{x - 3}$

$-2(x - 3) = 3(y - 2)$

$-2x + 6 = 3y - 6$

$\underline{\underline{2x + 3y = 12}}$

ii.



Since line AB meets A & B on coordinate axes; It will satisfy the given equation.

<u>For point A</u>:

$2x + 3y = 12$

$2x + 3(0) = 12$

$2x = 12$

$\underline{\underline{x}} = \underline{\underline{6}}$

A (x, 0) = $\underline{\underline{(6, 0)}}$

Since AB meets the y – axis at point B, abscissa of B is O.

<u>For point B</u>:

$2x + 3y = 12$

$2(0) + 3y = 12$

$3y = 12$

$y = \frac{12}{3}$

$\underline{\underline{Y}} = \underline{\underline{4}}$

B (0, y) = $\underline{\underline{(0, 4)}}$

From the diagram,

x-intercept = 6

y-intercept = 4

∴ Area of A $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times 4$

= $\underline{\underline{\text{12 sq. Units}}}$